Math 445 Homework 6 (revised)

Due Wednesday, October 27

26. Show that if p is an odd prime and a is a primitive root mod p, then $\left(\dfrac{a}{p}\right) = -1$.

27. [Pepin's Theorem] Show that the Fermat number $F_n = 2^{2^n} + 1$, for $n \geq 1$, is prime $\Leftrightarrow 3^{\frac{F_n - 1}{2}} \equiv -1 \pmod{F_n}$.

28. The primes p for which $x^2 \equiv 13 \pmod{p}$ has solutions consists precisely of those primes lying in certain congruence classes mod 13 ; which ones?

    [Hint: if you think of the classes as being represented by $-6, \dots, 0, \dots, 6$ then you can recycle a lot of your work....]

29. [NZM, p. 148, # 3.3.15] Show that if $p \geq 7$ is an odd prime, then $\left(\dfrac{n}{p}\right) = \left(\dfrac{n+1}{p}\right)$ for at least one of $n = 2, 3$, or 8.

    [Hint: it might help to express this in terms of $\left(\dfrac{n}{p}\right)\left(\dfrac{n+1}{p}\right)$

30. Compute $\left(\dfrac{35}{149}\right)$, $\left(\dfrac{39}{145}\right)$, and $\left(\dfrac{280}{351}\right)$.