Math 445 Homework 1

Due Wednesday, Sept. 8

1. (NZM, Problam 1.3.27) Show that if n is *not* prime, then $n|(n-1)!$.

2. (NZM, Problam 1.3.32) Show that for $n > 1$, $n^4 + 4$ is *never* prime.
(Hint: $f(x) = x^4 + 4$ can be expressed as a product of quadratics; find the factorization!)

3. (NZM, Problem 2.1.27) Show, by induction, that $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$ is an integer for every $n \geq 1$. (Note, however, that it is *not* a multiple of n !)

4. Show, by induction on n that
$$[\text{for every integer } x \geq 1,\ n! \text{ divides } x(x+1)\cdots(x+n-1).]$$
(Hint: prove the statement in brackets $[\cdots]$ by induction on x !)

5. Show that if $a \geq 2$ and $a^n - 1$ is prime, then n must be prime.