Math 445

Take-home Exam (Exam 2)

Due in class on Friday, November 19. You are not to discuss the exam, except on trivial matters, with anyone other than the instructor, until after you have turned in your solutions. The problems are given in approximately the order in which the material was presented in class; this is not necessarily a recommendation to work the problems in that order. Each needed computation should be carried out in full.

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Show that if $p \equiv 1 \pmod 4$, p is prime, and $a^2 + b^2 = p$ with $a > 0$ and odd, then $\left(\dfrac{a}{p}\right) = 1$.

   (Hint: $\left(\dfrac{p}{a}\right)$ makes sense, as a Jacobi symbol, and we can compute it....)

2. (20 pts.) Show (by induction on n) that if $x = [a_0, a_1, \ldots, a_n, \ldots] > 1$, then $\dfrac{1}{x} = [0, a_0, a_1, \ldots, a_n, \ldots]$. Show how to describe the convergents $\dfrac{H_n}{K_n}$ of $\dfrac{1}{x}$ in terms of the convergents $\dfrac{h_n}{k_n}$ of x .

3. (20 pts.) Show that the equation $x^2 = 2 + 41y$, $x, y \in \mathbb{Z}$ has a solution, but the equation $x^2 = 2 + 41y^2$, $x, y \in \mathbb{Z}$ has no solution.

4. Show that if $n \equiv 3 \pmod 4$, then the length of the periodic part of the continued fraction of $\sqrt{n}$ is *even*.

   (Hint: this has something to do with $x^2 - ny^2 =$*something* ...)

5. (20 pts.) Find a solution to the equation $x^2 - 29y^2 = 7$, $x, y \in \mathbb{Z}$, with $x \geq 1000$ and $y \geq 1000$.