Math 445

Some problems from an old Exam 1

You will be directed to **Show all work.** How you get your answer is just as important, if not more important, than the answer itself. If you think it, you should write it!

1. Find the period of the repeating decimal expansion of $1/41$ (by computing the order of the appropriate integer modulo the appropriate integer).

2. Show that if $ab \equiv 1 \pmod{n}$, then $ord_n(a) = ord_n(b)$.

3. Find the number of (incongruent, modulo 21) solutions to the congruence equation
$$x^5 \equiv 4 \pmod{21}$$

4. Show that if an integer n can be expressed as the sum of the squares of two *rational* numbers
$$n = (\frac{a}{b})^2 + (\frac{c}{d})^2 \ ,$$
then n can be expressed as the sum of the squares of two *integers*.

And a few more, for your amusement:

I. Use induction to show that, for every integer $n \geq 0$, the number
$$23 \cdot (49^n) \, - \, 6 \cdot (15)^n$$
is always a multiple of 17 .

II. Use the Euclidean Algorithm to find the greatest common divisor of
131 and 457
Then reverse your calculations to express $(131, 457)$ as a combination of 131 and 457.

III. Find the number of solutions to each of the congruence equations
(a) $x^2 \equiv 7 \pmod{75}$
(b) $x^4 \equiv 5 \pmod{17^2}$