Math 417 Problem Set 4

Starred (*) problems are due Friday, February 19.

27. (Gallian, p.72, #49) If G is a group with $a, b \in G$, so that $|a| = 4$, $|b| = 2$, and $a^3b = ba$, find the value of $|ab|$.

(*) 28. If G is a group with $a, b \in G$, and $ab = b^2a$ and $a^2b = ba$, show that $a = b = e$.

[What <u>other</u> "words" in a and b are equal to one another?]

(*) 29. (Gallian, p.87, #14) Suppose that G is a <u>cyclic</u> group that has exactly three subgroups: G, $\{e\}$, and a subgroup of order 7. What is $|G|$? Is there anything special about the number 7 ?

30. (Gallian, p.88, #24, sort of) Show that if G is a group with $a, b \in G$ and $ab = ba$, then $\langle b \rangle \leq C_G(a) =$ the centralizer of a in G.

31. (Gallian, p.89, #31) If G is a <u>finite</u> group, show that there is an integer $n \geq 1$ so that $a^n = e$ for all $a \in G$.

[The smallest such n is called the *exponent* of the group G, and will divide any other value of n (Why?).]

32. (Gallian, p.98, #38) If G is a group and $a, b \in G$ have $|a^2| = |b^2|$, must we have $|a| = |b|$? Show it is always true, or give an example of a group where it is false!

33. If G is a group and $a, b \in G$ have $|a| = 12$ and $|b| = 15$, then what are all of the possible orders of the subgroup $H = \langle a \rangle \cap \langle b \rangle$? You can arrange for each of the possibilities to occur by choosing appropriate elements of a single group $G = (\mathbb{Z}_n, +, 0)$; show how!

(*) 34. Show that if is G is a group and $a, b \in G$ with $|a| = 5$ and $|b| = 7$, then $\langle a \rangle \cap \langle b \rangle = \{e\}$. Use this to show that if, in addition, G is abelian, then $|ab| = 35$.