Math 417 Problem Set 2

Starred (*) problems are due Friday, February 5.

9. (Gallian, p.24, #19) Show that $\gcd(n, ab) = 1$

$$\text{if and only if } \gcd(n, a) = 1 \text{ and } \gcd(n, b) = 1.$$

[This is what 'makes' $\mathbb{Z}_n^*$ a group under multiplication; the product of two numbers relatively prime to n is a number relatively prime to n.]

10. Use the Euclidean algorithm to find the inverses of the elements 2, 3, and 7 in the group $G = (\mathbb{Z}_{137}^*, \cdot, 1)$.

11. (Gallian, p.55, #11) Find the inverse of the element $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL_2(\mathbb{Z}_{11})$.

(*) 12. Find the inverse of the element $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 5 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ in $GL_3(\mathbb{Z}_7)$.

[Hint: do what you would do in Math 314! 'Division' will be multiplication by inverses mod 7, instead...]

13. (Gallian, p.57, #42) Suppose that $F_1 = F(\theta)$ and $F_2 = F(\psi)$ (to adopt Gallian's notation) are reflections in lines of slope θ and ψ, with $\theta \neq \psi$, and $F_1 \circ F_2 = F_2 \circ F_1$. Show that then $F_1 \circ F_2 = R(\pi)$ is rotation by angle π.

[Your results from Problem #1 might help!]

(*) 14. (Gallian, p.57, #34) Prove that if G is a group and $a, b \in G$ then $(ab)^2 = a^2b^2$ if and only if $ab = ba$.

15. Give an <u>example</u> of a group G and $a, b \in G$ so that $(ab)^4 = a^4b^4$, but $ab \neq ba$.

[Hint: Problem #13 might help? Slightly bigger challenge: try the same thing with the 4's replaced by 3's !]

(*) 16. (Gallian, p.69, # 4) Show that if G is a group and $a \in G$, then $|a| = |a^{-1}|$.

17. If G is a group and $a \in G$, and if $|a| < \infty$ and $\gcd(k, |a|) = 1$, show that then $|a^k| = |a|$.