Math 417 Problem Set 5

Starred (*) problems are due Friday, September 28.

(*) 28. (Gallian, p.88, #24, sort of) Show that if G is a group with $a, b \in G$ and $ab = ba$, then $\langle b \rangle \leq C_G(a)$ = the centralizer of a in G.

29. Show that if G is a group, $A, b \in G$ and $ab \in Z(G)$ [the center of G], then $ab = ba$ (i.e., a and b commute).

(*) 30. (Gallian, p.86, #17) If $a \in G$ and $|a| < \infty$, then complete the following statement:

"$|a^2| = |a^{12}|$ if and only if ---------- ."

Explain why your statement is true.

31. (Gallian, p.87, #14) Suppose that G is a <u>cyclic</u> group that has exactly three subgroups: G, $\{e\}$, and a subgroup of order 7. What is $|G|$? Is there anything special about the number 7 ?

32. (Gallian, p.112, #3) Write each of the following permutations as a product of disjoint cycles:

(a) (1 2 3 5)(4 1 3)

(b) (1 3 2 5 6)(2 3)(4 6 5 1 2)

(c) (12)(1 3)(2 3)(1 4 2)

33. (Gallian, p.114, #32) If $\beta = (1\ 2\ 3)(1\ 4\ 5)$, express β^{99} as a product of disjoint cycles.

(*) 34. Show that if $\alpha \in S_n$ has $|\alpha|$ <u>odd</u>, then α is an <u>even</u> permutation!

[Hint: Imagine that you have expressed α as a product of disjoint cycles...]