Math 417 Problem Set 1

Starred (*) problems are due Friday, Augsut 31.

(*) 1. Some linear algebra(!) shows that that rotation $R(\theta)$ by angle θ around the origin, and reflection $M(\theta)$ in the line through the origin making angle θ, are linear transformations, given in matrix terms as multiplication by

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \text{ and } M(\theta) = \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$$

Using matrix multiplication, show that $M(\theta) \circ M(\psi)$ is a rotation, and $M(\theta) \circ R(\psi)$ and $R(\psi) \circ M(\theta)$ are both reflections, and determine which angle they rotate or reflect by.

[You can show yourself that the matrices are correct, since their columns are the vectors that $R(\theta)$ and $M(\theta)$ send the 'standard' basis vectors of $\mathbb{R}^2$ to.]

(*) 2. (Gallian, p.38, #14) If we build a rhombus R (a quadrilateral with all four sides having equal length) by gluing two equilateral triangles together along a pair of sides, describe the symmetries of R in terms of rotations and reflections.

3. Describe the symmetries of a (right circular) cylinder (of height h and radius r).

4. Show that the set $G = \{1, 5, 9, 13\}$ forms a group, with group multiplication being multiplication modulo 16. (One approach: build the 'Cayley' table! This helps to see why some needed properties hold.)

(*) 5. (Gallian, p.55, #18) Which elements $x \in D_4$ = the group of symmetries of a regular 4-gon (i.e., square) satisfy $x^2 = e$? Which satisfy $x = y^2$ for some $y \in D_4$?

[Problem #1 can help you decide what an element y^2 can look like...]

6. (Gallian, p.57, #48) Show that the collection of all 3×3 matrices

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

with $a, b, c \in \mathbb{R}$ forms a group under matrix multiplication.

[This group is known as the *Heisenberg group*, and arises in the study of one-dimensional quantum systems. You may find your row reduction prowess useful in finding inverses!]