Math 325 Problem Set 7

Due Wednesday, March 28

24. ("An accumulation point of the set of accumulation points is an accumulation point.") Show that, for a subset D of $\mathbb{R}$, if b_i, for $i \in \mathbb{N}$ are all accumulation points of D, and if $b_i \to b$ as $i \to \infty$, then b is also an accumulation point of D. [Note: it is enough to show that for every $n \in \mathbb{N}$ there is a $c_n \in D$ with $c_n \neq b$ and $|c_n - b| < \frac{1}{n}$; you will probably need to have a little care to make sure that the c_n you build, based on what we are given, is not equal to b !]

25. [Lay, p.198, problem # 20.13] (The Squeeze Play Theorem for functions): Show that *if* $f, g, h : D \to \mathbb{R}$ are functions, c is an accumulation point of D, $f(x) \leq g(x) \leq h(x)$ for all x in D and $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, *then* $\lim_{x\to c} g(x) = L$.

26. [Lay, p.198, problem # 20.15] Show that if $f, g : D \to \mathbb{R}$ are functions and c is an accumulation point of D, and if $\lim_{x\to c} f(x) = 0$ and $|g(x)| \leq M$ for all $x \in D$ and some constant M, then $\lim_{x\to c}(fg)(x) = 0$.

27. [Lay, p.208, problem # 21.10] If $f : D \to \mathbb{R}$ is a function which is continuous at $c \in D$, then the function $g = |f| : D \to \mathbb{R}$ defined as $g(x) = |f(x)|$ is also continuous at c. Show by example, however, that the opposite is not true: it is possible for $|f|$ to be continuous at c while f itself is discontinuous at c.