Math 325 Problem Set 3

Due Friday, February 10

8. [Lay, p.127, # 12.8] If S and T are subsets of $\mathbb{R}$ with $S \subseteq T$ and T is bounded both above and below, then show that S is also bounded, and
$$\inf(T) \leq \inf(S) \leq \sup(S) \leq \sup(T) \ .$$

9. [Lay, p.104, # 10.8] Show that for every $n \geq 1$ we have
$$\sum_{k=1}^{n} \frac{1}{4k^2 - 1} = \frac{n}{2n+1}$$
[One way: Factor $4k^2 - 1$!]

10. [Lay, p.105, # 10.22] Use induction to establish *Bernoulli's Inequality*: If $x \in \mathbb{R}$ and $x + 1 > 0$, then for every $n \in \mathbb{N}$ we have $(x+1)^n \geq 1 + nx$.

11. [Lay, p.106, # 10.26] Show that for every $n \in \mathbb{N}$, there is a $k \in \mathbb{N}$ so that $n \leq k^2 \leq 2n$.

    [N.B.: I'm guessing you can do this by induction (on n)... Or you can ask: how far apart are $\sqrt{n}$ and $\sqrt{2n}$?]