Math 325 Problem Set 2

Due Wednesday, February 1

4. [Lay, p.115, # 11.5] We define $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$. Show that for every pair of real numbers $x, y \in \mathbb{R}$, $|x| \cdot |y| = |xy|$.

5. [Lay, p.127, # 12.6(a)] Show that the least upper bound of a set S is unique; that is, if S is bounded from above, and if α and β both satisfy the properties required so be the supremum of S, then $\alpha = \beta$.

6. [Lay, p.127, # 12.3,12.4(g,h)]
Find the supremum (= lub) and infimum (= glb) of each of the following sets:

   (α) $\left\{\frac{n}{n+1} \ : \ n \in \mathbb{N}\right\}$

   (β) $\left\{(-1)^n(1+\frac{1}{n}) \ : \ n \in \mathbb{N}\right\}$

7. For subsets $A, B \subseteq \mathbb{R}$, we define their 'sum' $A + B = \{a + b \ : \ a \in A, b \in B\}$.
Show that if A and B are both bounded from above, then
$$\text{lub}(A + B) = \text{lub}(A) + \text{lub}(B) \ .$$
[Hint: show that lub(A)+lub(B) is an upper bound! Then worry about whether there might be a smaller one...]