Name:

Math 325, Section 1

Exam 3

Due on the (instructor's) desk at the start of class Wednesday, April 18. Your answers on the exam must represent your own work; do not consult with anyone other than your instructor, except on trivial matters, about the questions on the exam, until after the due date of the exam. The exam consists of five (5) questions on five (5) pages, numbered one through five; please verify that your copy of the exam conforms to these parameters. Each problem is work 20 points. If additional space is needed, please firmly attach any additional pages used.

1. Show that every *subsequence* $(a_{n_k})_{k=1}^{\infty}$ of a *monotonic* sequence $(a_n)_{n=1}^{\infty}$ is also monotonic.

2. Show, by example, that it is possible for a function $f : D \to \mathbb{R}$ to be continuous, for a number a to be an accumulation point of D, but the limit $\lim_{x \to a} f(x)$ <u>does</u> <u>not</u> <u>exist</u>.

3. Show that if $f : [0, 2] \to \mathbb{R}$ is *continuous* and $f(0) = f(2)$, then there is a(t least one) $c \in [0, 1]$ satisfying $f(c) = f(c + 1)$.

[Hint: construct a second function that you can apply the intermediate value theorem to, to get the conclusion that we want!]

4. Show that if $A, B, C \subseteq \mathbb{R}$ and the functions $f : A \to B$ and $g : B \to C$ are both *uniformly continuous*, then the composition $g \circ f : A \to C$ [defined by $(g \circ f)(x) = g(f(x))$] is also uniformly continuous.

5. Show that if $f, g : [a, b] \to \mathbb{R}$ are a pair of bounded functions, and $U(h)$ denotes the upper Riemann integral of h over the interval $[a, b]$, then

$$U(f + g) \leq U(f) + U(g) \; .$$