Math 325 Elementary Analysis
List of topics covered

Final Exam Monday April 30, 3:30pm-5:30pm, Oldfather 208

The Real Numbers

the real numnbers; the axioms of a field
natural numbers, integers, rationals, reals
ordering; triangle inequality
least upper bounds, greatest lower bounds; completeness
the Archimedean property; mathematical induction

Sequences

sequences, convergence (ϵ-N)
arithmetic operations on sequences $(+,-,\cdot,/)$, $\leq$
monotone sequences; bounded implies convergent!
Cauchy sequences; Cauchy implies convergent!
subsequences; every sequence has a monotone subsequence!
limsup, liminf; convergent $\Leftrightarrow$ liminf = limsup

Limits of functions

definition of limit, ϵ-δ proofs
accumulations points; uniqueness of limits
finding limits using sequences; nonexistence of limits using sequences!
arithmetic operations and limits $(+,-,\cdot,/)$, $\leq$

Continuity

continuity at a point, relation to sequences
continuity and $(+,-.\cdot,/)$, composition
Intermediate value theorem; root finding
Extreme value theorem; max/min
uniform continuity; unif cts implies cts!

Integration

partitions, upper and lower Riemann sums $U(f,P)$, $L(f,P$
Riemann integrability; partition with small mesh implies $U(f,P)-L(f,P)$ small
continuous implies integrable; monotone implies integrable
Fundamental Theorem of Calculus

Uniform Convergence

derivatives; mean value theorem
intermediate value theorem for derivatives
failures of continuity: holes, jumps, no one-sided limit
pointwise limit of a sequence of functions; Baire class one functions
uniform convergence
continuity, uniform continuity, integrability are preserved by uniform convergence
Weierstrass M-test
  application: power series
radius of convergence
term-by-term integration
term-by-term differentiation
continuous functions that are differentiable nowhere!