Math 325 Problem Set 6

Starred (*) problems are due Friday, October 12.

(*) 31. (Belding and Mitchell, p.80, #2) Show that if $f : (a, b) \to \mathbb{R}$ is a continuous function, then the function $g : (a, b) \to \mathbb{R}$ given by $g(x) = |f(x)|$ is also continuous. (You should argue directly from ϵ's and δ's.)

32. Using the previous problem (and a problem from a previous problem set!), show that if $f, g : (a, b) \to \mathbb{R}$ are continuous functions, then the function $M : (a, b) \to \mathbb{R}$ given by $M(x) = \max\{f(x), g(x)\}$ is also continuous.

(*) 33. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = 0$ for every $x \in \mathbb{Q}$. Show that $f(x) = 0$ for every $x \in \mathbb{R}$.

[Hint: what's the alternative? Remember that rational numbers are 'everywhere'!]

34. Using the problem above, show that if $f, g : \mathbb{R} \to \mathbb{R}$ are both continuous functions, and $f(x) = g(x)$ for every $x \in \mathbb{Q}$, then $f = g$ (i.e., $f(x) = g(x)$ for every $x \in \mathbb{R}$).

['A continuous function is <u>determined</u> by its values on the rational numbers.']

35. Suppose that $a < 0 < b$ and $f : (a, b) \to \mathbb{R}$ is a function that is bounded (i.e., for some $M \in \mathbb{R}$, $|f(x)| \le M$ for every $x \in (a, b)$). Show that the function $g : (a, b) \to \mathbb{R}$ defined by $g(x) = xf(x)$ is continuous at $x = 0$. Show, on the other hand, that for any <u>other</u> $c \in (a, b)$ we have that g is continuous at c <u>if</u> <u>and</u> <u>only</u> <u>if</u> f is continuous at c.

[The last assertion can be attacked using 'general' results we have established, <u>or</u> directly using ϵ's and δ's (your choice!).]

(*) 36. (Belding and Mitchell, p.89, #9) Use the intermediate value theorem to show that any positive number $a \in \mathbb{R}$, $a > 0$ has an n-th root, that is, for any $n \in \mathbb{N}$, there is some real number $x \ge 0$ such that $x^n = a$.

[The textbook provides an outline that you could follow.]

37. Show that if $f : [a, b] \to [a, b]$ is continuous, then there is at least one $c \in [a, b]$ so that $f(c) = c$.

[Hint: apply the intermediate value theorem to a different function!]