Math 325 Problem Set 2

Starred (*) problems are due Friday, September 7.

(*) 5. For the statements (a)-(c) below, state both the contrapositive and the 'proof by contradiction' form "it is not possible to have both..." versions of the given statement, and indicate (no explanation needed) which of the resulting collections of statements are <u>true</u>.

(a) If $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.

(b) If $a \notin \mathbb{Q}$, then $\dfrac{1}{a} \notin \mathbb{Q}$.

(c) If $a \notin \mathbb{Q}$ and $b \notin \mathbb{Q}$, then $ab \notin \mathbb{Q}$.

(*) 6. Show, using the Rational Roots Theorem, that $\alpha = \sqrt{2 + \sqrt{3}}$ is not rational.

[Find a polynomial with integer coefficients that has α as a root!]

7. Show, using the Rational Roots Theorem, that $\beta = 2^{1/2} + 5^{1/3}$ is not rational.

[Hint: start by raising $\beta - \sqrt{2}$ to the third power...!]

(*) 8. By looking at the first few cases, find a (short) formula for the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)} \ ;$$

then prove, using induction, that your formula is correct.

9. Show, using induction, that for every $n \in \mathbb{N}$ we have

$$\sum_{k=1}^{n} k(k+1)(k+2) = \frac{1}{4}n^4 + \frac{3}{2}n^3 + \frac{11}{4}n^2 + \frac{3}{2}n \ .$$

[Hint: going straight at it is fine; a 'slicker' way is to note that the quartic <u>is</u>

$$\frac{1}{4}n(n+1)(n+2)(n+3) \ ...]$$