Math 325 Elementary Analysis
Checklist of topics covered

Final Exam Wednesday, December 12, 1:00pm-3:00pm, Burnett 204

The Real Numbers

The real numbers; the axioms of a field
Natural numbers, integers, rationals, reals
Ordering on the real line; triangle inequality
Least upper bounds, greatest lower bounds; completeness
Archimedean Principle; Rational Roots Theorem
Proof techniques: mathematical induction; proof by contradiction

Limits of functions and Continuity

Definition of limit, ϵ-δ proofs; uniqueness of limits
Arithmetic operations and limits $(+,-,\cdot,/)$, $\leq$ (squeeze play theorem)
Continuity at a point, continuity on an interval
Continuity and arithmetic operations $(+,-\cdot,/)$, composition
Intermediate value theorem; root finding
Extreme value theorem; maxs/mins
Uniform continuity; uniformly cts implies cts!
Heine-Borel Theorem; continuity on a closed interval
Inverse Function Theorem (continuity of inverse functions)

Differentiation

Difference quotient; differentiability; differentiable implies continuous
Differentiation of sums, differences, products, quotients, compositions
The 'other' inverse function theorem; Mean Value Theorem; applications
Cauchy Mean Value Theorem; L'Hôpital's Rule; Taylor polynomials

Integration

Partitions; Riemann sums $R(f,P)$; upper and lower Riemann sums $U(f,P)$, $L(f,P)$
Riemann integrability; $\|P\| < \delta$ implies $|R(f,P) - L| < \epsilon$
Alternate formulations: $\|P\| < \delta$ implies $|U(f,P) - L(f,P)| < \epsilon$
$P \subseteq Q$ implies $U(f,Q) \leq U(f,P)$, $L(f,P) \leq U(f,Q)$
Continuous implies integrable; monotone implies integrable
Sum of integrable fcns is integrable; integrable on a <u>sub</u>interval
Fundamental Theorem<u>s</u> of Calculus; continuous fcns have antiderivatives
f integrable and has antiderivative F implies $\int_a^b f(x)\,dx = F(b) - F(a)$
Intermediate value theorem for derivatives; not every fcn has an antiderivative!
Using antiderivatives to define $\ln x$, e^x
Polynomial approximations; Taylor polynomials;
The remainder as an integral; error estimates

Uniform Convergence

Pointwise limit of a sequence of functions; failure to preserve 'nice' properties
Uniform convergence; continuity, integrability are preserved by uniform convergence
Consequences for Taylor series: term-by-term integration and differentiation