Math 314: Things we know how to do

Solve a system of equations $A\vec{x} = \vec{b}$ using row reduction/Gaussian elimination.

Show that a linear system has no solutions, one solution, many solutions (check for consistency, count free variables).

Show that a collection of vectors in $\mathbb{R}^n$ span $\mathbb{R}^n$ (pivot in every row), show they are linearly independent (no free variables).

Balance a chemical equation.

Compute the net flow through a network, by monitoring (i.e., knowing the value at) some of the edges.

Compute the matrix for a linear transformation; compute the image of a vector under a transformation.

Compute the inverse of a matrix A (using a super-augmented matrix).

Write an (invertible) matrix as a product of elementary matrices (by keeping track of the row operations in reducing it to the identity matrix).

Compute the solution to a system of equations $A\vec{x} = \vec{b}$ with A invertible (by inverting!).

Compute the determinant of a matrix (by row reduction, or by expanding along row/column).

Determine if a collection of vectors form a vector space/subspace (check closure under addition, scalar multiplication).

Interpret linear systems in terms of column spaces and nullspaces (column = who has solutions, null = how many).

Express a column space as a nullspace (of another matrix), and the same in reverse.

Find bases for column space, row space, nullspace (row reduce!).

Compute the rank and nullity of a matrix.

Start with linearly independent vectors in a subpace, extend to a basis (add a basis at the end, then row reduce, keep the columns corresponding to pivots).

Start with a spanning set for a subspace, choose a basis (row reduce, keep columns corresponding to pivots).

Find the coordinates of a vector with respect to a basis.

Analyze a Markov chain; find the steady state solution that all initial states converge to.

Compute the characteristic polynomial of a matrix.

Compute the eigenvalues and bases of eigenspaces for a matrix.

Diagnonalize a matrix, or show that it cannot be done (geometric vs. algebraic multiplicity). Use diagonalization to compute "high" powers of a matrix.

Build a basis for the orthogonal complement of a subspace (described as span? $(\mathrm{col}(A))^\perp = \mathrm{null}(A^T)$. described as nullspace? $(\mathrm{null}(A))^\perp = \mathrm{row}(A)$.)

Use $(\mathrm{col}(A))^\perp = \mathrm{null}(A^T)$ to build a test for consistency of a system $A\vec{x} = \vec{b}$ ($\vec{b}$ must be $\perp$ every vector in a basis for $\mathrm{null}(A^T)$).

Build an orthogonal (orthonormal) basis for a subspace (start with a basis, and apply Gram-Schmidt).

Compute the orthogonal projection of a vector to a subspace (build an orthogonal basis, and sum the projections onto each basis vector [or see below!]).

Decompose a vector $\vec{v}$ into the sum of a vector $\vec{w} \in W$ and $\vec{w}' \in W^\perp$.

Find the vector in $\mathrm{col}(A)$ closest to $\vec{b}$, i.e., find $\vec{x}$ so that $\|A\vec{x} - \vec{b}\|$ is as small as possible (solve $A^T A\vec{x} = A^T\vec{b}$, take $A\vec{x}$). [This is the same as taking the orthogonal projection of $\vec{b}$ onto $\mathrm{col}(A)$.]

Find the line which best fits a collection of data points.

Find a degree k polynomial whose graph best fits a collection of data points.