Name:

Math 314/814 Matrix Theory

Exam 1

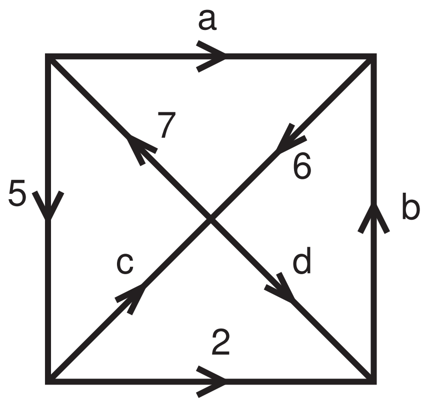
Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Show whether or not the system of linear equations given by the augmented matrix

$$(A|\mathbf{b}) = \left(\begin{array}{cccc|c} 1 & 2 & 4 & 1 & 2 \\ 3 & 1 & 7 & -6 & -3 \\ 2 & 5 & 9 & 4 & 7 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution?

2. (20 pts.) The figure below models a network of pipes, with numbers indicating the flow rate (in the direection of each arrow) on some pipes, and variables indicating unknown flow rates. Are the known rates sufficient to determine all of the remaining rates? If so, determine the unknown rates; if not, what further values would be sufficient to determine the remaining rates?



3. (25 pts.) Is the vector $\vec{b} = \begin{bmatrix} 7 \\ 21 \\ 1 \end{bmatrix}$ in the span of the vectors

$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$? Are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly independent?

4. (20 pts.) Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\mathrm{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 2 & 7 & 4 \end{pmatrix}$$

(b) (5 pts.) Use your answer from part (a) to find the solution to the equation $\mathrm{A}x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

5. (10 pts.) Find an example of matrices A and B so that

$A\vec{u} = \vec{0}$ and $B\vec{v} = \vec{0}$ both have non-trivial solutions,

but $(A+B)\vec{w} = \vec{0}$ has no non-trivial solution.

[Hint: your matrices don't need to be very big...]