**Math 314/814**

**Things we know how to do**

- Solve a system of equations $A\vec{x} = \vec{b}$ using Gaussian elimination.
- Show that a linear system has no solutions, one solution, many solutions (check for consistency, count free variables).
- Compute the rank and nullity of a matrix.
- Show that a collection of vectors in $\mathbb{R}^n$ span $\mathbb{R}^n$ (pivot in every row), show they are linearly independent (no free variables).
- Balance a chemical equation.
- Compute the net flow through a network, by monitoring (i.e., knowing the value at) some of the edges.
- Show that a matrix is invertible.
- Write an (invertible) matrix as a product of elementary matrices (by keeping track of the row operations in reducing it to the identity matrix).
- Compute the inverse of a matrix $A$ (using a super-augmented matrix).
- Determine if a collection of vectors form a subspace (check closure under addition, scalar multiplication).
- Interpret linear systems in terms of column and nullspaces (column = who has solutions, null = how many).
- Find bases for column space, row space, nullspace (row reduce!).
- Find the coordinates of a vector with respect to a basis for a subspace.
- Start with linearly independent vectors in a subpace, extend to a basis (add a basis at the end, then row reduce, keep the columns corresponding to pivots).
- Start with a spanning set for a subspace, choose a basis (row reduce, keep columns corresponding to pivots).
- Compute the matrix for a linear transformation; compute the image of a vector under a transformation.
- Compute the transition matrix of a Markov chain; find the steady state solution for the modeled system (via eignvector/eigenvalue).
- Compute the determinant of a matrix (by row reduction, or by expanding along row/column).
- Compute the solution to a system of equations $A\vec{x} = \vec{b}$ with $A$ invertible (by inverting! or Cramer's rule). Compute the inverse of a matrix (using minors).
- Compute the characteristic polynomial of a matrix.
- Compute the eigenvalues and bases of eigenspaces for a matrix.
- Diagnonalize a matrix, or show that it cannot be done (geometric vs. algebraic multiplicity). Use diagonalization to compute "high" powers of a matrix.
- Show two matrices *aren't* similar (by showing they have different eigenvalues, or characteristic polynomials, or geometric multiplicities).
- Build a basis for the orthogonal complement of a subspace (described as span? $(\text{col}(A))^\perp = \text{null}(A^T)$. described as nullspace? $(\text{null}(A))^\perp = \text{row}(A)$.)
- Use $(\text{col}(A))^\perp = \text{null}(A^T)$ to build a test for consistency of a system $A\vec{x} = \vec{b}$ ($\vec{b}$ must be $\perp$ every vector in a basis for $\text{null}(A^T)$).
- Build an orthogonal (orthonormal) basis for a subspace (start with a basis, and apply Gram-Schmidt).
- Compute the orthogonal projection of a vector to a subspace (build an orthogonal basis, and sum the projections onto each basis vector [or see below!]).
- Decompose a vector $\vec{v}$ into the sum of a vector $\vec{w} \in W$ and $\vec{w}' \in W^\perp$.
- Find the vector in $\text{col}(A)$ closest to $\vec{b}$, i.e., find $\vec{x}$ so that $\|A\vec{x} - \vec{b}\|$ is as small as possible (solve $A^TA\vec{x} = A^T\vec{b}$, take $A\vec{x}$). [This is the same as taking the orthogonal projection of $\vec{b}$ onto $\text{col}(A)$.]
- Find the line which best fits a collection of data points.
- Find the degree $k$ polynomial whose graph best fits a collection of data points.