Math 314/814 Matrix Theory
Exam 1 Practice probrlams

Show all work. Include all steps necessary to arrive at an answer unaided by a mechanical computational device. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. Use row reduction to find a solution to the following system of linear equations:

$$\begin{array}{rcrcrcr} -x & & & + & 2z & = & 3 \\ x & - & y & - & 3z & = & 3 \\ 2x & + & 3y & - & 3z & = & 6 \end{array}$$

2. Use row reduction to decide if the system of linear equations given by the augmented matrix:

$$(A|\mathbf{b}) = \left(\begin{array}{cccc|c} -1 & 0 & 2 & 3 & 2 \\ 3 & 2 & -4 & 1 & -2 \\ 0 & 3 & 0 & 1 & 6 \end{array}\right)$$

has a solution. If it does, does it have one or more than one solution?

3. Is the vector $\vec{b} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$ in the span of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$?

More generally, what equation among a, b, c must hold in order for $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to be in the span of $\vec{v}_1, \vec{v}_2$?

4. Use Gauss-Jordan elimination to find the inverse of the matrix A, where

$$\mathrm{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 5 & 8 \end{pmatrix}$$

(b) Use your answer from part (a) to find the solution to the equation $\mathrm{A}x = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

5. Let $\mathbf{O}$ denote the $n \times n$ matrix with all entries equal to 0.

Suppose that A and B are $n \times n$ matrices with

$$AB = \mathbf{O} \ ,$$

but

$$B \neq \mathbf{O} \ .$$

Show that A **cannot** be invertible.
(Hint: suppose it *is*: what does that tell you about B ?)