Math 314/814, Section 6

Quiz number 6 Solutions

Suppose that for an $n \times n$ matrix A, there is a target vector $\vec{b}$ so that the equation $A\vec{x} = \vec{b}$ has <u>no</u> solution. Explain why we can conclude that A <u>cannot</u> be invertible.

There were many different solutions among those given by the class. Here are some.

Since $A\vec{x} = \vec{b}$ has no solution, the augmented matrix $(A|\vec{b})$ after row reducing has a row of zeros facing a non-zero number. In particular, A row reduces to a matrix with a row of zeros, i.e., A does not have a pivot in every row. So it cannot be invertible; an invertible matrix row reduces to I_n, so has a pivot in every row.

Or: same argument as above, but not every column has a pivot (because not every row does, and the matrix is square!).

Or: since $A\vec{x} = \vec{b}$ has no solution, $\vec{b}$ is not a linear combination of the columns of A, so the columns of A do not span all of $\mathbb{R}^n$. But if A <u>is</u> invertible its columns must span $\mathbb{R}^n$. So A can't be invertible.

Or: If A were invertible, then A^{-1} exists, so $A^{-1}A = I_n$. But then $\vec{x} = A^{-1}(A\vec{x}) = A^{-1}\vec{b}$ <u>is</u> a solution to $A\vec{x} = \vec{b}$, since $A(A^{-1}\vec{b}) = (AA^{-1})\vec{b} = I_n\vec{b} = \vec{b}$. But this wasn't supposed to happen! So A can't be invertible.