Name:

Math 314 Section 6

Quiz number 4 Solution

In one model of an economy, there are four sectors, A, B, C, D, which each purchase the output of the others:

The production of A uses 30% of the resources for each of B, C, D.
Production of B uses no D, and uses 40% of A and C.
Production of C uses no A, and uses 20% of B and D.
Production of D requires 20% of A, and 30% of each of B and C.

Finally, the remaining fraction of each sector's output is used as a resource for that sector.

Based on this information, find the system of equations that will determine the equilibrium prices A, B, C, D of each commodity, so that the revenues generated equal the cost of the resources for each commodity, and express this system as an augmented matrix. **Do not solve this system** (unless you are really bored...)**.**

The matrix which describes the economy has rows which list the fraction of each sector's output used by the sector corresponding to the row. The information, therefore, fits into each row. What is missing above is what fraction of each sector is used by that same sector. But the other sectors use $40\% + 0\% + 20\% = 60\%$ of A, leaving 40%. They use $30\% + 20\% + 30\% = 80\%$ of B, leaving 20%. They use $30\% + 40\% + 30\% = 100\%$ of C, leaving 0%. Finally, they use $30\% + 0\% + 20\% = 50\%$ of D, leaving 50%.

So the matrix describing how output is distributed is given by

$$\begin{pmatrix} 0.4 & 0.3 & 0.3 & 0.3 \\ 0.4 & 0.2 & 0.4 & 0.0 \\ 0.0 & 0.2 & 0.0 & 0.2 \\ 0.2 & 0.3 & 0.3 & 0.5 \end{pmatrix}$$

Because we want revenues, P_A, P_B, P_C, P_D to equal expenses, this matrix represent the equations

$$\begin{aligned} P_A &= 0.4P_A + 0.3P_B + 0.3P_C + 0.3P_D \\ P_B &= 0.4P_A + 0.2P_B + 0.4P + C + 0.0P_D \\ P_C &= 0.0P_A + 0.2P_B + 0.0P_C + 0.2P_D \\ P_D &= 0.2P_A + 0.3P_B + 0.3P_C + 0.5P_D \end{aligned}$$

which becomes

$$\begin{aligned} &-0.6P_A + 0.3P_B + 0.3P_C + 0.3P_D = 0 \\ &0.4P_A - 0.8P_B + 0.4P + C + 0.0P_D = 0 \\ &0.0P_A + 0.2P_B - 1.0P_C + 0.2P_D = 0 \\ &0.2P_A + 0.3P_B + 0.3P_C - 0.5P_D = 0 \end{aligned}$$

This becomes the augmented matrix

$$\left(\begin{array}{cccc|c} -0.6 & 0.3 & 0.3 & 0.3 & 0 \\ 0.4 & -0.8 & 0.4 & 0.0 & 0 \\ 0.0 & 0.2 & -1.0 & 0.2 & 0 \\ 0.2 & 0.3 & 0.3 & -0.5 & 0 \end{array}\right)$$