Math 314 Section 6

Quiz number 3 Solution

Show that the vector $\begin{bmatrix}3\\0\\0\\0\end{bmatrix}$ is **not** in the span of the vectors $\begin{bmatrix}1\\3\\2\\2\end{bmatrix}$, $\begin{bmatrix}2\\7\\4\\1\end{bmatrix}$, and $\begin{bmatrix}1\\1\\1\\2\end{bmatrix}$.

(Show that it cannot be expressed as a linear combination....)

The problem asserts that the vector equation

$$(*) \qquad x\begin{bmatrix}1\\3\\2\\2\end{bmatrix} + y\begin{bmatrix}2\\7\\4\\1\end{bmatrix} + z\begin{bmatrix}1\\1\\1\\2\end{bmatrix} = \begin{bmatrix}3\\0\\0\\0\end{bmatrix}$$

has no solution. That is, the system of equations with matrix

$$\begin{pmatrix}1 & 2 & 1 & | & 3\\ 3 & 7 & 1 & | & 0\\ 2 & 4 & 1 & | & 0\\ 2 & 1 & 2 & | & 0\end{pmatrix}$$

is inconsistent. This is something that we can establish by row reduction:

$$\begin{pmatrix}1 & 2 & 1 & | & 3\\ 3 & 7 & 1 & | & 0\\ 2 & 4 & 1 & | & 0\\ 2 & 1 & 2 & | & 0\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & -2 & | & -9\\ 2 & 4 & 1 & | & 0\\ 2 & 1 & 2 & | & 0\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & -2 & | & -9\\ 0 & 0 & -1 & | & -6\\ 2 & 1 & 2 & | & 0\end{pmatrix} \longrightarrow$$

$$\begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & -2 & | & -9\\ 0 & 0 & -1 & | & -6\\ 0 & -3 & 0 & | & -6\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & -2 & | & -9\\ 0 & 0 & -1 & | & -6\\ 0 & 1 & 0 & | & 2\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & 0 & | & 2\\ 0 & 0 & -1 & | & -6\\ 0 & 1 & -2 & | & -9\end{pmatrix} \longrightarrow$$

$$\begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & 0 & | & 2\\ 0 & 0 & 1 & | & 6\\ 0 & 1 & -2 & | & -9\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & 0 & | & 2\\ 0 & 0 & 1 & | & 6\\ 0 & 0 & -2 & | & -11\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & 1 & 0 & | & 2\\ 0 & 0 & 1 & | & 6\\ 0 & 0 & 0 & | & 1\end{pmatrix}$$

This last matrix is in row echelon form (REF), which is good enough to check consistency. The last row, however, represents the equation "$0 = 1$", which no assignment of values to the variables x, y, z can make true. So the system of equations is inconsistent; there is no solution to the vector equation (*). So we cannot write our target vector as a linear combination of the three vectors given; the target does not lie in the span of those three vectors.