Math 310 Homework 9

Due Tuesday, November 20

42. Find the values of x which solve the system of equations

$$x \equiv 1 \pmod{4}$$
$$x \equiv 2 \pmod{7}$$
$$x \equiv 3 \pmod{9}$$
$$x \equiv 4 \pmod{13}$$

43. If R and S are rings show that $(r, s) \in R \times S$ is

    (a) an idempotent $\iff$ r and s both are

    (a) nilpotent $\iff$ r and s both are.

44. If R is a ring, show that

$$S = \{(r, r) : r \in R\} \subseteq R \times R$$

    is a subring of $R \times R$, and $S \cong R$.

45. Show by induction that if $\varphi : R \to S$ is a homomorphism, $x \in R$, and $n \in \mathbb{N}$, then

    (a) $\varphi(n \cdot x) = n \cdot \varphi(x)$

    (b) $\varphi(x^n) = (\varphi(x))^n$

46. Show that $\mathbb{Z}_2 \times \mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_4$ are not isomorphic. Show, however, that the two rings have the same number of units, zero divisors, idempotents, and nilpotent elements!

    (Hint (for the first part): where must $4 \cdot (1, 1)$ be sent, under a homomorphism from $\mathbb{Z}_2 \times \mathbb{Z}_8$ to $\mathbb{Z}_4 \times \mathbb{Z}_4$?)