Math 310 Homework 8

Due Tuesday, November 13

38. Show that if $R \cong S$ and R is an integral domain, then so is S.

39. If R is a ring with $0_R \neq 1_R$, then an element $a \in R$ cannot be *both* a zero divisor *and* a unit.

40. Show that the ring $\mathbb{Z}_5[i] = \{a + bi : a, b \in \mathbb{Z}_5\}$, with addition and multiplication defined as in problem 36, is *not* a field.

41. Show that "is isomorphic to" is an equivalence relation, i.e, for any three rings R, S, and T,

    (a) $R \cong R$

    (b) If $R \cong S$, then $S \cong R$

    (c) If $R \cong S$ and $S \cong T$, then $R \cong T$

    (Hint: the "obvious" functions work, but don't forget to show that each is both bijective *and* a homomorphism!)

For Math 310H, or extra credit:

H5. Let n be a positive integer that is *not* the square of another integer (so that $\sqrt{n}$ is not rational). Let

$$\mathbb{Q}[\sqrt{n}] = \{a + b\sqrt{n} : a, b \in \mathbb{Q}\} \subseteq \mathbb{R}$$

with the usual addition and multiplication from $\mathbb{R}$. Show that $\mathbb{Q}[\sqrt{n}]$ is a *subfield* of $\mathbb{R}$, i.e., it is both a subring and a field in its own right.