**Math 310 Homework 4**

Due Tuesday, October 2

18. (Childs, p.67, E2 (sort of)) Show that if $a$ is an *odd* integer and $ab \equiv ac \pmod 8$, then $b \equiv c \pmod 8$ .

    What are some other numbers besides 8 for which this will work?

19. Show by induction (on $n$) that if the integers $a_1, \ldots, a_n$ are all congruent to 1 modulo $m$, then their product $a_1 \cdot \cdots \cdot a_n \equiv 1 \pmod m$ .

20. (Childs, p.67, E4) Show that if $a \equiv b \pmod m$ and $n$ is any natural number, then $a^n \equiv b^n \pmod m$ .

21. (Childs, p67, E6) Find the remainder of each of the numbers $a$ when you divide by the corresponding number $b$ :

    (1): $a = 5^{18}$ , $b = 7$

    (2): $a = 68^{105}$, $b = 13$

    (3): $a = 6^{47}$ , $b = 12$

    (Hint: Problem 20, suitably applied, will help!)

22. Show that if $a$ and $b$ are integers with $a \equiv b \pmod p$ for *every* prime $p$, then $a = b$ . (Hint: How big is $|b - a|$ ?)

**For Math 310H, or extra credit:**

H3. Show that every integer of the form $n = 4m + 3$ has a prime factor of the form $4k + 3$. (Translation: every number that leaves remainder 3 upon division by 4 has a prime factor that leaves remainder 3 upon division by 4.) Use this to show that there are infinitely many prime numbers of the form $4m + 3$ .

Hint (for both parts!): What is the alternative? Mimic our proof of the infinitude (what a wonderful word....) of primes, for the second part....