Math 310 Homework 3

Due Tuesday, September 25

13. Use the Euclidean Algorithm to find the gcd of $a = 1111$ and $b = 473$, then reverse the steps of your calculations to write $\gcd(a, b)$ as a combination of a and b.

14. Repeat problem 13, with the numbers $a = 1357$ and $b = 2468$.

15. (Childs, p.49, E1) Show by induction that if a prime number p divides the product $a_1 \cdot \cdots \cdot a_n$ of n integers $a_1, \ldots, a_n$, then p divides at least one of the a_i.

16. Show that if a is an integer, $n \geq 1$, p is prime, and $p|a^n$, then $p^n|a^n$.

17. (Childs, p.50, E3) Show that if $n \geq 1$ is *not* prime, then n can be factored as $n = pq$ where p is prime and $p \leq \sqrt{n}$. Use this to determine whether or not 239 is prime.

For Math 310H, or extra credit:

H2. (Childs, p.51, E5) Show that if a and b are integers, both ≥ 1 and with $(a, b) = 1$, and $ab = c^r$, then $a = x^r$ and $b = y^r$ for some integers x and y.

(Hint: Ignore Childs' hint, he was trying to be too clever. One approach is to use complete induction (on c).)