Math 310 Homework 2

Due Tuesday, September 18

7. Show by induction that for any integers a and b, and integer $k \geq 0$
$$a + b \text{ divides } a^{2k+1} + b^{2k+1}$$
Conclude that for any integer $a \geq 2$ and odd integer $k \geq 3$, $a^k + 1$ is *never* prime.
(Hint: this is really very much like Problems 2 and 3, just with no actual numbers...)

8. Show that if $n \geq 1$ and $2^n + 1$ is a prime number, then n must be a power of 2 (i.e., every prime factor of n is 2!).
(Hint: Suppose not! Problem 7 will help....)

9. (Childs, p.26, E1) Show that for any three consecutive integers $n, n+1, n+2$, *exactly one* of them is divisible by 3.

10. Show that if $a|c$ and $b|d$, then $ab|cd$.

11. Show that if $a|b$ and $a|c$, then $a|rb + sc$ for all integers r and s.

12. Show that if $a|c$ and $b|c$, and $(a, b) = 1$, then $ab|c$
(Hint: write $1 = an + bm$, then mutliply by c and stare at the right-hand side of the equation.....)

For Math 310H, or extra credit:

H1. Show that if $a|(b + c)$ and $(b, c) = d$, then $(a, b) \leq d$ *and* $(a, c) \leq d$. Give an example where the inequalities are *not* actually equalities!