

Name:

Math 221, Section 3

Quiz number 4

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. In order to make learning Math 221 more "cool", you decide to refrigerate your textbook before reading it. If you place your 75° book into your 35° fridge and, 5 minutes later, you find it has reached a temperature of 65° , how much **longer** will you need to wait before your book reaches a comfortable reading temperature of 50° ?

$$T(t) = \text{temp at time } t \quad T(0) = 75 \quad T(5) = 65 \quad A = 35$$

When \nexists $T(t) = 50$?

$$T' = k(A - T) \quad T' + kT = kA$$

$$\begin{aligned} T(t) &= e^{-\int k dt} \left(\int e^{sk dt} kA dt + C \right) \\ &= e^{-kt} \left(\int (ke^{kt}) A dt + C \right) = e^{-kt} (Ae^{kt} + C) \\ &= A + Ce^{-kt} = 35 + C e^{-kt} \quad T(0) = 75 = 35 + C, C = 40 \\ T(t) &= 35 + 40e^{-kt} \end{aligned}$$

$$T(5) = 65 = 35 + 40e^{-5k}, 30 = 40e^{-5k}, e^{-5k} = \frac{3}{4}$$

$$-5k = \ln\left(\frac{3}{4}\right), k = -\frac{1}{5} \ln\left(\frac{3}{4}\right).$$

$$T(t) = 35 + 40 e^{-\frac{1}{5} \ln\left(\frac{3}{4}\right) t} = 35 + 40 e^{\ln\left(\left(\frac{3}{4}\right)^{\frac{t}{5}}\right)} = 35 + 40 \left(\frac{3}{4}\right)^{\frac{t}{5}}$$

$$50 = T(t) = 35 + 40 \left(\frac{3}{4}\right)^{\frac{t}{5}} \Rightarrow \left(\frac{3}{4}\right)^{\frac{t}{5}} = \frac{50-35}{40} = \frac{15}{40} = \frac{3}{8}$$

$$\rightarrow \frac{1}{5} \ln\left(\frac{3}{4}\right) = \ln\left(\left(\frac{3}{4}\right)^{\frac{t}{5}}\right) = \ln\left(\frac{3}{8}\right), \boxed{t = \frac{5 \ln\left(\frac{3}{8}\right)}{\ln\left(\frac{3}{4}\right)}}$$

So $\frac{5 \ln\left(\frac{3}{8}\right)}{\ln\left(\frac{3}{4}\right)} \sim 5 \text{ minutes later, book is } 50^\circ.$