Name: Solution

Math 221, Section 3

Quiz number 11

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Use Laplace tansforms to find the solution to the initial value problem

$$y'' + 2y' + 5y = 0$$
$$y(0) = 2 \text{ , } y'(0) = 0$$

$$0 = \mathcal{L}\{0\} = \mathcal{L}\{y'' + 2y' + 5y\}$$
$$= \mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 5\mathcal{L}\{y\}$$
$$= s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 2(s\mathcal{L}\{y\} - y(0)) + 5\mathcal{L}\{y\}$$
$$= (s^2+2s+5)\mathcal{L}\{y\} - 2s - 0 + 2(-2)$$

So need $\mathcal{L}\{y\} = \dfrac{2s+4}{s^2+2s+5} = \dfrac{2s+4}{(s+1)^2+4} = \dfrac{2(s+1)+2}{(s+1)^2+(2)^2}$

So $y = \mathcal{L}^{-1}\left\{\dfrac{2(s+1)+2}{(s+1)^2+(2)^2}\right\} = 2\mathcal{L}^{-1}\left\{\dfrac{s+1}{(s+1)^2+2^2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{2}{(s+1)^2+2^2}\right\}$

$$= 2\mathcal{L}^{-1}\left\{\frac{s-(-1)}{(s-(-1))^2+2^2}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s-(-1))^2+2^2}\right\}$$

$=$ (from the tables) $2e^{-t}\cos(2t) + e^{-t}\sin(2t)$

So $y = 2e^{-t}\cos(2t) + e^{-t}\sin(2t)$.