Name:

Math 221, Section 5

Quiz number 6

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Find a linearly independent pair of solutions to the homogeneous differential equation

$$(*) \qquad y'' + 2y' + y = 0$$

and a particular solution to the corresponding inhomogeneous differential equation

$$(**) \qquad y'' + 2y' + y = \sin(3x) \ .$$

[N.B.: both methods we learned will work, but one is less of a headache than the other...]

Use these to describe the general solution to the differential equation (**).

The characteristic equation $a^2 + 2a + 1 = (a+1)^2 = 0$ has roots $a = -1$ and $a = -1$, and so a fundamental set of homogeneous solutions is $y_1 = e^{-x}$ and $y_2 = xe^{-x}$.

Since $y = \sin(3x)$ is not a solution to the homogeneous equation, the method of undetermined coefficients implies that a particular solution to (**) is given by

$y = c_1 \sin(3x) + c_2 \cos(3x)$. To determine the constants, we compute:

$y' = 3c_1 \cos(3x) - 3c_2 \sin(3x)$ and
$y'' = -9c_1 \sin(3x) - 9c_2 \cos(3x)$, so
$y'' + 2y' + y$
$= [-9c_1 \sin(3x) - 9c_2 \cos(3x)] + 2[3c_1 \cos(3x) - 3c_2 \sin(3x)] + [c_1 \sin(3x) + c_2 \cos(3x)]$
$= [-9c_1 - 6c_2 + c_1] \sin(3x) + [-9c_2 + 6c_1 + c_2] \cos(3x)$
$= [-8c_1 - 6c_2] \sin(3x) + [6c_1 - 8c_2] \cos(3x)$,

so we need $-8c_1 - 6c_2 = 1$ and $6c_1 - 8c_2 = 0$. So $c_1 = \frac{8}{6}c_2 = \frac{4}{3}c_2$,
and $-8\frac{4}{3}c_2 - 6c_2 = -\frac{50}{3}c_2 = 1$, so $c_2 = -\frac{3}{50}$. Then $c_1 = \frac{4}{3}(-\frac{3}{50}) = -\frac{4}{50}$.
So $y_p = -\frac{4}{50}\sin(3x) - \frac{3}{50}\cos(3x)$ is a particular solution.

So the general solution to the equation (**) is

$$y = c_1 e^{-x} + c_2 x e^{-x} - \frac{4}{50}\sin(3x) - \frac{3}{50}\cos(3x)$$