Math 221 Sample Final Exam

Show all work. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. (10 pts.) Show that both of the functions

$$x(t) = 2t^{\frac{1}{2}} - 3 \qquad \text{and} \qquad x(t) = -3$$

are solutions to the initial value problem

$$2tx' - x = 3 \qquad x(0) = -3$$

Why doesn't this violate our existence/uniqueness theorem for IVPs?

2. (10 pts.) Find the solution to the initial value problem

$$y' = \frac{3}{t}y + t^2 \qquad y(1) = 9$$

3. A set of fundamental solutions to the Cauchy-Euler equation

$$x^2y'' + 4xy' + 2y = 0 \ .$$

is $y = x^{-1}$ and $y = x^{-2}$.

(b): (10 pts.) Use variation of parameters to find the solutions to the inhomogeneous equation

$$x^2y'' + 4xy' + 2y = e^x \ .$$

4. (15 pts.) Find the general solution to the differential equation

$$y''' + 2y' - 3y = \sin t$$

5. (15 pts.) Find the general solution to the system of equations

$$\begin{aligned} x' &= -4x - 9y - t \\ y' &= x + 2y \end{aligned}$$

6. (15 pts.) Find the function whose inverse Laplace transform is the solution to the initial value problem

$$y'' - y' + y = \begin{cases} t & \text{if } 0 \le t \le 3 \\ 3 & \text{if } t > 3 \end{cases}$$

$$y(0) = 1 \quad , \quad y'(0) = 2$$

7. (15 pts.) Use Laplace tansforms to find the solution to the initial value problem

$$y' - 3y = 5t$$
$$y(0) = 2$$