Math 221 Section 5

Exam 2 Practice Problems

Exams provide you, the student, with an opportunity to demonstrate
your understanding of the techniques presented in the course. So:

Show all work. The steps you take to your answer are just as important, if not more important, than the answer itself. If you think it, write it!

1. (15 pts.) Find the solution to the initial value problem

$$2y'' - 7y' + 3y = 0$$

$$y(0) = 1 \quad , \quad y'(0) = -7$$

2. (15 pts.) One solution to the differential equation

$$y'' - \frac{1}{t}y' + \frac{1-t}{t}y = 0$$

is $y_1(t) = e^t$.

Use reduction of order to find a second, independent, solution to the equation.

3. (15 pts.) The functions

$$y_1(t) = t \text{ and } y_2(t) = te^t$$

form a fundamental set of solutions to the differential equation

$$y'' - \frac{t+2}{t}y' + \frac{t+2}{t^2}y = 0 \ .$$

Use variation of parameters to find a particular solution to the equation

$$y'' - \frac{t+2}{t}y' + \frac{t+2}{t^2}y = t^3 \ .$$

4. (20 pts.) Find the general solution to the differential equation

$$y''' - 2y'' + 2y' - y = 2e^t - \sin t$$

5. (20 pts.) Find the solution to the initial value problem

$$x' = 2x - 9y \qquad x(0) = 1$$
$$y' = x + 2y \qquad y(0) = -1$$

6. (15 pts.) Sketch the direction field for the system of equations

$$x' = 2xy - xy^2$$
$$y' = x - y$$

labelling all nullclines and equilibrium solutions.