Math 208H, Section 1

Practice problems for Exam 2

A1. Find the local extrema of the function

$$f(x, y) = 2x^4 - 2xy + y^2 \ ,$$

and determine, for each, if it is a local max. local min, or saddle point.

A2. Find the point(s) on the ellipse $g(x, y) = x^2 + 3y^2 = 4$ where the function $f(x, y) = x - 3y + 4$ achieves it maximum value.

A3. Evaluate the iterated integral $\displaystyle\int_0^2 \int_x^2 x^2(y^4 + 1)^{1/3}\, dy\, dx$ by rewriting the integral to reverse the order of integration. (Note: the integral *cannot* be evaluated in the order given....)

A4. Find the integral of the function $f(x, y, z) = x + y + z$ over the region lying between the graph of $z = x^2 + y^2 - 4$ and the x-y plane.

A5. Find the integral of the function $f(x, y) = xy^2$ over the region lying in the first quadrant of the x-y plane and lying inside of the circle $x^2 + y^2 = 9$.

A6. Find the integral of the function $f(x, y) = 6x + y^2$ over the region in the x-y plane between the x-axis and the lines $y = x$ and $y = 6 - 2x$.

A7. Find the integral of the function $f(x, y) = xy^2$ over the region in the plane lying between the graphs of $a(x) = 2x$ and $b(x) = 3 - x^2$.

A8. Evaluate the following double integrals:

(a): $\displaystyle\int_0^1 \int_1^2 x^2y - y^2x \ \mathrm{d}x\ \mathrm{d}y$ (b): $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 x\sqrt{y}\ \mathrm{d}y\ \mathrm{d}x$

A9. Find the integral of the function $f(x, y) = x$ over the region R lying between the graphs of the curves

$$y = x - x^2 \text{ and } y = x - 1.$$

A10. Use Lagrange multpliers to find the maximum value of the function $f(x, y) = xy$ subject to the constraint $g(x, y) = x^2 + 4y^2 - 1 = 0$.

A11. Find the area of the region S bounded by one loop of the curve described by $r = \sin(3\theta)$ in polar coordinates. (Hint: to determine the limits of integration, when is $r = 0$?)

A12. A particle is moving through 3-space along the parametrized curve $\vec{r}(t) = (\cos t, \sin t, t^{3/2})$. Find:

(a) the velocity of the particle at time t,

(b) the acceleration of the particle at time t, and

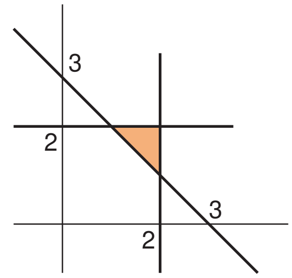
(c) the length of the curve traced out by the particle between $t = 0$ and $t = 2$.

A13. Find the critical points of the function $f(x,y) = 2xy^2 - x^2 - 8y^2$,
and for each, determine if the point is a rel max, rel min, or saddle point.

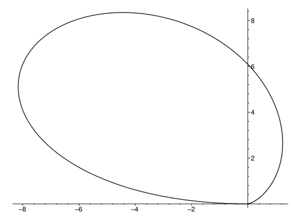
A14. Find the point(s) on the graph of the equation $g(x,y) = x^2 + 4y^2 = 8$
that **minimizes** the function $f(x,y) = x + 2y$.

A15. Find the integral of the function $f(x,y) = \dfrac{x}{y}$
over the region in the plane lying between the lines $x + y = 3$, $x = 2$, and $y = 2$.

[Note: one iterated integral is probably less trouble than the other; which variable would you prefer to integrate first?]



A16. Recall that the area of a region R in the plane can be computed as the integral of the function $f(x,y) = 1$ over the region. Use this, and polar coordinates, to find the area of the region lying inside of the *polar* curve
$r = \theta^2(\pi - \theta)$, $0 \le \theta \le \pi$ (see figure)



A17. Sketch the region involved, and set up, *but do not evaluate*, an iterated integral which will compute the integral of the function

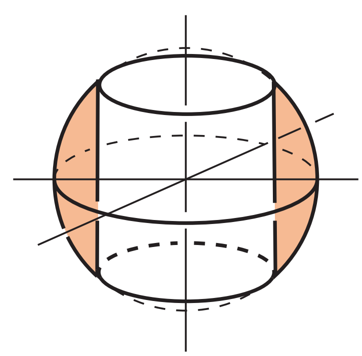
$$f(x,y,z) = xy + z$$

over the region in 3-space lying in the first octant ($x \ge 0$, $y \ge 0$, $z \ge 0$) and below the plane

$$\frac{x}{2} + \frac{y}{4} + \frac{z}{5} = 1$$

A18. Set up, *but do not evaluate*, the integrals, in **both** cylindrical and spherical coordinates, which will compute the
integral of the function $f(x,y,z) = x + 3y$
over the region in 3-space lying *inside* of the
sphere $x^2 + y^2 + z^2 = 9$ of radius 3,
and *outside* of the cylinder $x^2 + y^2 = 4$ of radius 2; see figure.

[Note: at least one of your answers will involve the arcsin of a number we do not know the arcsin of....]



A19. Find the integral of the vector field $F(x,y) = (xy, x + y)$
along the parametrized curve $\vec{r}(t) = (e^t, e^{2t})$ $0 \le t \le 1)$.