Name: Solution

Math 208H, Section 1

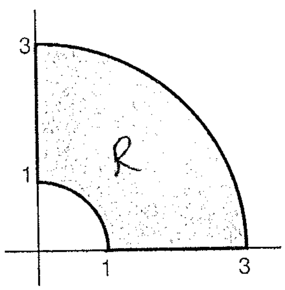
Quiz number 7

The y-coordinate $\overline{y}$ of the center of mass of the region R in the (x, y)-plane lying in the first quadrant between the circles of radius 1 and 3 centered at the origin (see figure) can be computed as

$$[\int_R y\, dA] \,/\, [\int_R 1\, dA]$$

(where $\int_R 1\, dA$ is the area of R, expressed as an integral). **Find** $\overline{y}$ **.**

[Note: The area of R <u>can</u> be obtained from the areas of the circles of radii 1 and 3, if you prefer that to the integral...]



$\cancel{0\le r}$ $1 \le r \le 3$

$0 \le \theta \le \frac{\pi}{2}$

$$\int_R y\, dA = \int_0^{\frac{\pi}{2}} \int_1^3 r\sin\theta \; \underline{r\, dr\, d\theta}$$

$$= \int_0^{\pi/2} \int_1^3 r^2 \sin\theta\, dr\, d\theta = \int_0^{\pi/2} \sin\theta \left.\frac{r^3}{3}\right|_1^3 d\theta =$$

$$= \int_0^{\pi/2} \sin\theta \left(\frac{27}{3} - \frac{1}{3}\right) d\theta = -\frac{26}{3}\cos\theta \Big|_0^{\pi/2} = -\frac{26}{3}\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= -26/3\,(0 - 1) = 26/3$$

$$\int_R 1\, dA = \int_0^{\pi/2} \int_1^3 r\, dr\, d\theta = \int_0^{\pi/2} \left.\frac{r^2}{2}\right|_1^3 d\theta = \int_0^{\pi/2} \frac{(9-1)}{2}\, d\theta$$

$$= \int_0^{\pi/2} 4\, d\theta = 4(\pi/2) = 2\pi$$

Area! $= \frac{1}{4}\left(\pi(3^2 - 1^2)\right)$

$= \frac{1}{4}(8\pi) = 2\pi$ ....

So $\overline{y} = \dfrac{(26/3)}{2\pi} = \boxed{\dfrac{13}{3\pi}}$