Math 208H, Section 1

Practice problems for Exam 2

9. Find the local extrema of the function

$$f(x,y) = 2x^4 - 2xy + y^2 \ ,$$

and determine, for each, if it is a local max. local min, or saddle point.

6. Find the point(s) on the ellipse $g(x,y) = x^2 + 3y^2 = 4$ where the function $f(x,y) = x - 3y + 4$ achieves it maximum value.

1. Evaluate the iterated integral $\displaystyle\int_0^2 \int_x^2 x^2(y^4+1)^{1/3}\, dy\, dx$ by rewriting the integral to reverse the order of integration. (Note: the integral *cannot* be evaluated in the order given....)

4. Find the integral of the function $f(x,y,z) = x + y + z$ over the region lying between the graph of $z = x^2 + y^2 - 4$ and the x-y plane.

3. Find the integral of the function $f(x,y) = xy^2$ over the region lying in the first quadrant of the x-y plane and lying inside of the circle $x^2 + y^2 = 9$.

5. Find the integral of the function $f(x,y) = 6x + y^2$ over the region in the x-y plane between the x-axis and the lines $y = x$ and $y = 6 - 2x$.

4. Find the integral of the function $f(x,y) = xy^2$ over the region in the plane lying between the graphs of $a(x) = 2x$ and $b(x) = 3 - x^2$.

5. Evaluate the following double integrals:

(a): $\displaystyle\int_0^1 \int_1^2 x^2y - y^2x \; \mathrm{d}x \; \mathrm{d}y$ (b): $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 x\sqrt{y} \; \mathrm{d}y \; \mathrm{d}x$

1. Find the integral of the function $f(x,y) = x$ over the region R lying between the graphs of the curves

$$y = x - x^2 \text{ and } y = x - 1.$$

5. Use Lagrange multpliers to find the maximum value of the function $f(x,y) = xy$ subject to the constraint $g(x,y) = x^2 + 4y^2 - 1 = 0$.

7. Find the area of the region S bounded by one loop of the curve described by

$r = \sin(3\theta)$

in polar coordinates. (Hint: to determine the limits of integration, when is $r = 0$?)

4. A particle is moving through 3-space along the parametrized curve $\vec{r}(t) = (\cos t, \sin t, t^{3/2})$. Find:

   (a) the velocity of the particle at time t,

   (b) the acceleration of the particle at time t, and

   (c) the length of the curve traced out by the particle between $t = 0$ and $t = 2$.