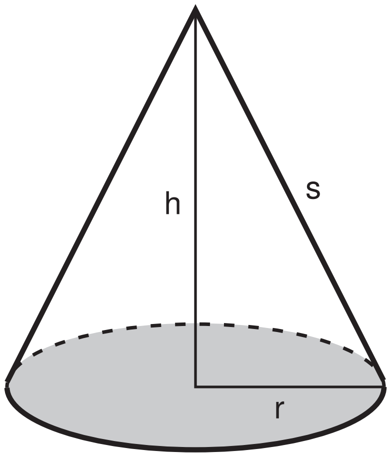
Introduction: You have been hired by McFee's Mathematical Circus to find the optimal design for their "big top" tent, utilizing 1000 square meters of canvas. There are two competing designs, and they wish to know the optimal choice (or choices) for each design. In the process of finding the answer, you will learn how to simplify optimization problems by adding extra constraints.

The first design is a cone:



Here, the volume of the tent is $V(r,h) = (\pi/3)r^2h$, while the (surface) area made from the canvas is $S(r,h) = \pi r\sqrt{r^2+h^2}$.

We will take two approaches to the problem:

First design, the "hard" way: Maximize $V(r,h)$, subject to the constraint that $S(r,h)$ = 1000 m^2. This you should solve as we have in class/homework/quiz/exam .

First design, the easier (?) way: Experience (and, no doubt, your solution to the above) teaches us that dealing with functions involving square roots can be difficult; so we will try to circumvent this difficulty by introducing an extra variable and a *second* constraint.

"How do you handle two constraints in a Lagrange multipliers problem?" you might ask.

The basic idea is that if you are trying to find extrema of the function $f(x,y,z)$ subject to the constraints $a(x,y,z) = C$ and $b(x,y,z) = D$ for some constants C and D, then you need to solve the equations

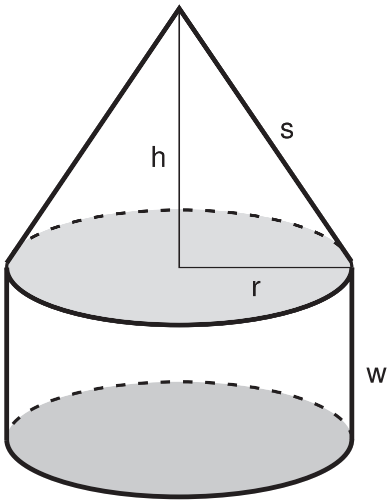
$$\nabla(f) = \lambda\nabla(a) + \mu\nabla(b), \quad a(x,y,z) = C, \quad b(x,y,z) = D$$

for the five variables x, y, z, λ, and μ.

In our example, we can introduce the *new* variable s, where $s^2 = r^2 + h^2$, and use this to simplify the function S. Use this method to find our optimal solution, by maximizing $V(r,h,s) = (\pi/3)r^2h$, subject to the constraints $S(r,h,s) = \pi rs$ = 1000 and $s^2 = r^2 + h^2$.

(HINT: With more variables and equations, it is important to be organized and methodically eliminate variables, starting with the simplest equations. You can try problem # 11 from section 14.3 of the textbook, as a warmup.)

Second design - any way you like: The second design you have been asked to optimize is a cylinder with a cone on top:



Again, find the values of h, r, and w that give the largest volume, given that you still have only 1000 m^2 of canvas from which to build the tent. You should choose whichever of the two methods we now have at our disposal that you wish to use, to solve this problem. (For the second method, we would recommend introducing the extra variable s depicted above.)

Write a report to the board of directors of McFee's Mathematical Circus, giving the optimal designs and explaining clearly how you obtained your results. Which of the two optimal designs would you recommend, and why? You can assume that the board has seen the concepts of Math 208 (but does not really remember them too clearly); make sure to include sufficient detail and explanatory material to make your work understandable. (If they can't understand it, how can they **pay** you?) Be sure to use complete sentences!

You may choose to work on this project individually, or to work with some of your fellow students, in groups of up to three in size. Each group need only turn in one project, headed by all of the names of your consulting team.

Your completed report is due on the board of directors' desk (or the desk of your Math 208 instructor, whichever is easiest to find) by noon on Monday, November 22.