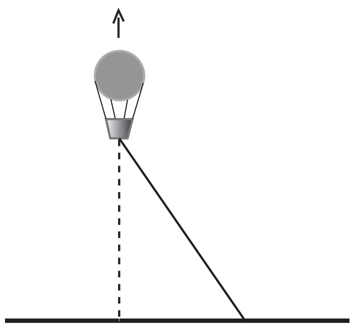
Name:

Math 1710 (old) Exam 2

Show all work (i.e., work things out on paper, not in your head).

1. A hot air balloon is rising vertically, at a constant rate of 5 feet per second. 50 feet from where the balloon took off, you are standing and holding a rope that is tied to the balloon (see figure). How fast is the rope passing through your hands when the balloon is 100 feet off of the ground? (In other words, how fast is the length of the rope between you and the balloon changing?) (25 pts.)



2. Let $f(x)$ be the function

$$f(x) = \frac{x^2+3}{x+1} = (x^2+3)(x+1)^{-1} = x - 1 + \frac{4}{x+1} = x - 1 + 4(x+1)^{-1}$$

(Note: these are all the **same** function!)

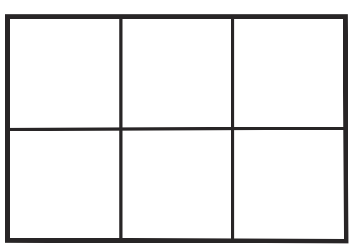
(a) Determine the intervals over which f is increasing and decreasing (12 pts.). (Note: one of the versions of the function is easier to differentiate than the others!)

(b) Determine the intervals over which f is concave up and concave down (12 pts.).

(c) Find all vertical asymptotes for f; determine the appropriate limits. Find the limits of $f(x)$ as $x \to \pm\infty$ (6 pts.)

(d) Using the information from (a)-(c), sketch a graph of the function $y = f(x)$ (5 pts.)

3. You are given 500 feet of fencing to build a rectangular pen, which will be subdivided into 6 pieces, by adding two vertical and one horizontal fence lines (see figure). What is the largest area that can be enclosed? (25 pts.)



Material for our exam ends here.

4. Use the tangent line to a graph, or differentials, to approximate the value of

$$(69)^{\frac{1}{3}} \text{ , knowing that } (64)^{\frac{1}{3}} = 4 \text{ . (15 pts.)}$$