Math 1650
Topics for first exam

Chapter 1: Functions and their graphs

§1: Graphs of equations
    Cartesian (x-y) plane
    graph = all points that *satisfy* the equation
    How to graph?
        plot points (and fill in gaps)
        use x- and y-intercepts
        use symmetry
            y-axis: (a,b) on graph, so is (-a,b)
            x-axis: (a,b) on graph, so is (a,-b)
            origin: (a,b) on graph, so is (-a,-b)
    Eqn for circle: $(x-h)^2+(y-k)^2=r^2$

§2: Lines and their slopes
    slope= rise over run = (change in y-value)/(corresponding change in x value)
    slope-intercept: $y=mx+b$
    point-slope: $\dfrac{y-y_0}{x-x_0}=m$
    two-point: $\dfrac{y-y_0}{x-x_0}=\dfrac{y_1-y_0}{x_1-x_0}$
    same slope: lines are parallel (do not meet)
    lines are perpendicular: slopes are **negative reciprocals**

§3: Functions
    function = rule which assigns to each input **exactly one** output
    inputs = domain; outputs = range/image; f:A→B
    y=f(x) : 'y equals f of x' : y equals the value assigned to x by the function f
    f,x,y, etc. are all placeholders; any other sybols are 'just as good'
    'implied' domain of f: all numbers for which f(x) *makes sense*

§4: Graphs of functions
    y=f(x) is an equation; graph the equation!
    graph = all pairs (x,f(x)) where x is in the domain of f
    all functions have graphs, but not all graphs 'have' functions
        function takes only one value at a point; vertical line test
    symmetry (for functions)
        y-axis: *even* function, f(-x) = f(x)
        x-axis: XXXXXX
        origin: *odd* function, f(-x) = -f(x)
    increasing on an interval: if $x>y$, then $f(x)>f(y)$
    decreasing on an interval: if $x>y$, then $f(x)<f(y)$
    constant

§5: Translations and combinations
    graph of y=f(x)
        shift to right by c; y=f(x-c)
        shift to left by c; y=f(x+c)
        shift down by c; y=f(x)-c
        shift up by c; y=f(x)+c
    y=af(x) ; stretch graph by factor of a
    reflect graph along axes
        y-axis: y=f(-x)
        x-axis: y=-f(x)
    combining functions: combine the outputs of two functions f,g
        f+g, f-g, fg, f/g
    composition: output of one function is input of the next
    f followed by g = g∘f; g∘f(x) = g(f(x)) = g of f **of** x

§6: Inverse functions
Idea: find a function that undoes f
find a function g so that g(f(x)) = x for every x
magic: f undoes g ! Usual notation: g = f^{-1}
Problem: not every function has an inverse.
need g to be a function; so f cannot take the same value twice.
horizontal line test
Graph of inverse: if (a,b) on graph of f, then (b,a) is on graph of f^{-1}
graph of f^{-1} is graph of f, reflected across line y=x

Chapter 2: Polynomials

§1: Quadratic functions
monomial = ax^n
polynomial = bunch of monomials = $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = f(x)$
$a_n \neq 0$, then n=*degree* of f
deg=0: constant fcn; deg=1: linear fcn; deg=2: *quadratic* fcn
$f(x) = ax^2 + bx + c$; graph = *parabola*
Standard form: $ax^2 + bx + c = a(x-h)^2 + k$
complete the square: $ax^2 + bx + c = a(x^2 + \frac{b}{a}x) + c$
add half of $\frac{b}{a}$, squared, inside parentheses
(and subtract corresponding amount outside!)
standard form →graph:
x^2 to $(x-h)^2$ (shift left/right) to
$a(x-h)^2$ (stetch/reflect) to $a(x-h)^2 + k$ (shift up/down)
lowest/highest point of graph = (h,k) = *vertex* of parabola
axis of symmetry: vertical line x=h

§2: General properties of polynomials
$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$; domain = everything
graph has no gaps, hole, or jumps (f is *continuous*)
can draw graph without lifting up writing implement
graph has no corners - no sudden turns; graph is *smooth*
behavior at 'ends':
n even, $a_n > 0$: high/high
n even, $a_n < 0$: low/low
n odd, $a_n > 0$: low/high
n odd, $a_n < 0$: high/low
root (zero) of f ; $f(a) = 0$; grpah of f hits x-axis at a
if $f(a) = 0$, then $f(x) = (x-a)g(x)$
nth degree polynomial can have at most n roots
nth degree polynomial can *turn around* at most (n-1) times
consequence of continuity: intermediate value theorem
if a polynomial takes on two values c and d, then
it also takes on every value in between
application: 'finding' roots: if $f(a) < 0$ and $f(b) > 0$, then
there is a root of f somewhere between a and b

§3: Polynomial division
root a of f ↔ factor $(x-a)$ of $f(x)$
reason: polynomial (long) division
$f(x) = (x-a)g(x) + b$; a=root, then b=0
polonomial = (divisor)(quotient) + remainder
degree of remainder is less than degree of divisor
synthetic division: fast method to divide by (x-a)