Math 107H

Final Exam Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. **Find any THREE (3) of the following FOUR (4) integrals**

1-1: $\displaystyle\int \sec^3 x \tan^3 x \; dx$

1-2: $\displaystyle\int \frac{x^2}{\sqrt{3-x^2}} \; dx$

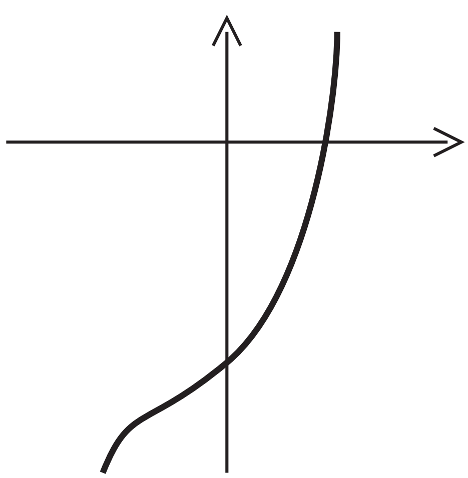
1-3: $\displaystyle\int x^2 e^{3x} \; dx$

1-4: $\displaystyle\int \frac{2x+3}{x^3+x^2-2} \; dx$

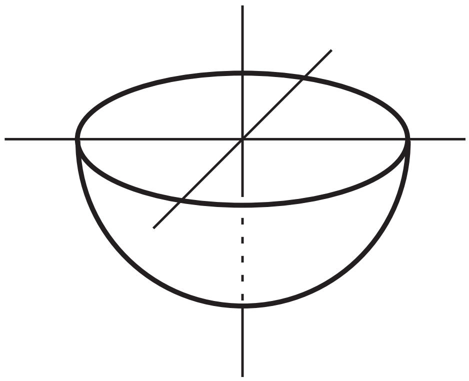
2. Find the area of the region lying between the graphs of the functions

$$f(x) = 2x - 1 \qquad \text{and} \qquad g(x) = x^4 + x - 1$$

3. Find the volume of the region obtained by revolving the region lying between the graph of $f(x) = x^3 + 7x - 22$ and the x- and y-axis around the line $x = -2$.



4. Find the work done in digging a hemispherical hole in the ground, having a depth of 12 meters. (That is, find the work required to lift the dirt to the surface; assume that dirt has a density of 300 kg per cubic meter.)



5. Find the following limits (10 pts. each):

(a): $\lim_{x\to\infty} \dfrac{x^2 - 3x^3 + 9}{4x^2 - 6x + 1}$

(b): $\lim_{x\to\infty} \dfrac{(x^2+1)^x}{(x+1)^{2x}}$

6. **Determine the convergence or divergence of any THREE (3) of the following FOUR (4) series** (5 pts. each)

6-1 $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^{1/2}}{n^2}$

6-2 $\displaystyle\sum_{n=2}^{\infty} \frac{n!}{(n^2+n-3)^{3/2}}$

6-3 $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n+3}{3n-5}\right)^n$

6-4 $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{5/3}}$

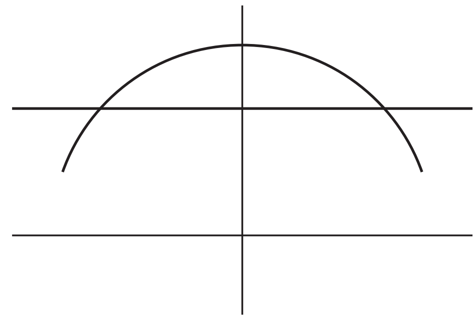
7. Find the degree three Taylor polynomial, centered at $c = 3$, for the function

$$f(x) = (x^2 - 5)^{5/2}$$

8. Find the arclength of the parametrized curve

$$x = t^4 \quad , \quad y = t^6$$
$$0 \le t \le 2$$

B-3. Find the area lying between the polar curves $r = 4$ and $r = \dfrac{2}{\sin(\theta)}$ (see figure).



B-4. For the integrals below, when the appropriate substitution is made, what (trigonometric) integral results?

(a) (10 pts.) $\displaystyle\int \frac{\sqrt{x^2-2}}{x^2}\,dx$

(b) (10 pts.) $\displaystyle\int \frac{x^2}{\sqrt{3-x^2}}\,dx$

B-6. Set up, <u>but</u> <u>do</u> <u>not</u> <u>solve</u> (because you can't!), an integral which will compute the length of the ellipse given by the equation

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$

[Hint: Finding a parametrization "close to" $x = \cos t$, $y = \sin t$ will help...]

B-9. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n}{n2^n + 3^n}(x-1)^n$$

B-10. Starting from the Taylor series for $f(x) = \dfrac{1}{1-x}$ centered at $a = 0$, show how to build the Taylor series for the function $g(x) = \frac{\ln(1+x^3)}{x}$ centered at $a = 0$.

[Hint: start by building $h(x) = \dfrac{1}{1+x}$ (!). Technical note: g is not defined at $x = 0$, but since $\lim_{x\to 0} g(x) = 0$, we can <u>pretend</u> that $g(0) = 0$ ...]

For problems 1 through 4, find the indicated integrals.

C-1. $\displaystyle\int (2x-3)^{5/2}\,dx$

C-2. $\displaystyle\int \frac{x}{(x+1)(x+3)}\,dx$

C-3. $\displaystyle\int_0^{\pi} x\sin(2x)\,dx$

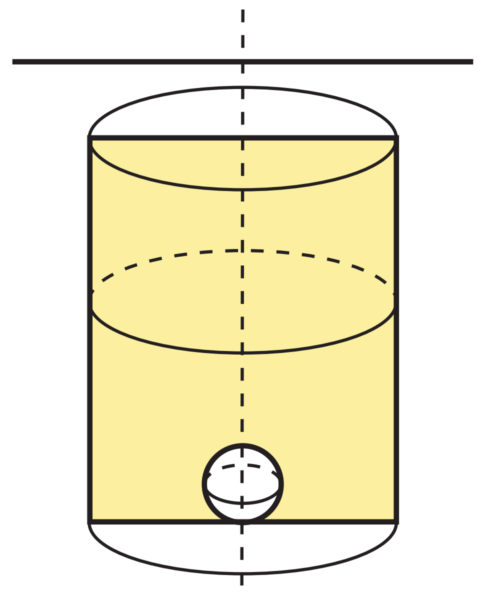
C-4. $\int \sqrt{x^2-1}\,dx$

C-5. Find the length of the parametrized curve

$$x = t^2 \quad , \quad y = t^3 \qquad \text{for } 0 \le t \le 2.$$

[Hint: This does <u>not</u> require trig substitution!]

C-6. A tank in the shape of a cylinder, with radius 4 meters and height 10 meters, is buried in the ground with its top 2 meters below the ground. A spherical stone, with radius 1 meter, rests on the bottom (see figure). Set up, **but do not evaluate**, the integral(s) needed to determine the work needed to pump the (otherwise) full tank of fluid (having density W kg/m^3) to ground level. [You will likely need to 'split' your integral 2 meters above the bottom of the tank!]



C-7. Determine whether or not the improper integral

$$\int_2^{\infty} \frac{\ln x}{x^2}\, dx$$

converges. What does this tell us about the series $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$?

C-8. Determine whether or not each of the following series converges.

(a): $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n (n+2)^2}$

(b): $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+1}$

C-9. For what values of x does the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n n^2}$ converge?

C-10. Find the Taylor polynomial, of degree 3, centered at $a = 1$, for the function

$$f(x) = xe^{-x} \ .$$