Math 107H, Section 1

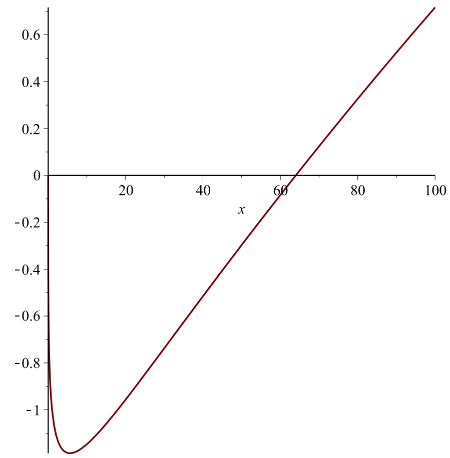
Quiz number 7 Solution

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

9. Find the volume obtained by revolving the region lying between the x-axis and the graph of the function

$$f(x) = x^{\frac{1}{2}} - 2x^{\frac{1}{3}} = x^{\frac{1}{3}}(x^{\frac{1}{6}} - 2)$$

around the x-axis (see figure).



The slice of the region will be a circle, from rotating the line segment from $f(x)$ to 0 around the x-axis, with radius $0 - f(x) = 2x^{\frac{1}{3}} - x^{\frac{1}{2}}$. The slice will then have area

$$A(x) = \pi(2x^{\frac{1}{3}} - x^{\frac{1}{2}})^2 = \pi(4(x^{\frac{1}{3}})^2 - 4x^{\frac{1}{3}}x^{\frac{1}{2}} + (x^{\frac{1}{2}})^2) = \pi(4x^{\frac{2}{3}} - 4x^{\frac{5}{6}} + x)$$

We also need to know where to start and end! This is where the graph of $y = f(x)$ crosses the x-axis, that is, where $x^{\frac{1}{3}}(x^{\frac{1}{6}} - 2) = 0$. This will happen when $x^{\frac{1}{3}} = 0$¡ so $x = 0$¡ and when $x^{\frac{1}{6}} - 2 = 0$, so $x^{\frac{1}{6}} = 2$¡ so $x = 2^6 = 64$.

So the volume of the region is:

$$\int_0^6 4\pi(4x^{\frac{2}{3}} - 4x^{\frac{5}{6}} + x)\, dx$$

$$= \pi(4 \cdot \frac{3}{5}x^{\frac{5}{3}} - 4 \cdot \frac{6}{11}x^{\frac{11}{6}} + \frac{1}{2}x^2)\Big|_0^{2^6}$$

$$= \pi(4 \cdot \frac{3}{5}(2^6)^{\frac{5}{3}} - 4 \cdot \frac{6}{11}(2^6)^{\frac{11}{6}} + \frac{1}{2}(2^6)^2) - \pi(0 - 0 + 0)$$

$$= \pi(\frac{12}{5}2^{10} - 4\frac{6}{11}2^{11} + \frac{1}{2}2^{12}$$

$$= \frac{\pi}{110}(4 \cdot 66 \cdot 2^{10} - 240 \cdot 2^{11} + 55 \cdot 2^{12})$$

$$= \frac{\pi}{110} \cdot 2^{12} \cdot (66 - 120 + 55)$$

$$= \frac{\pi}{110} \cdot 2^{12}$$

$$= \frac{\pi}{55} \cdot 2^{11}$$

although we probably all long since stopped caring about the exact value....