Math 107H, Section 1

Quiz number 2 Solutions

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

Find the following integrals.

2. $\displaystyle\int_0^{\pi/6} 2\sin x + \cos x \; dx$

We can find antiderivatives, and then evaluate at the endpoints. Since

$\displaystyle\int \sin x \; dx = -\cos x + C$ and $\displaystyle\int \cos x \; dx = \sin x + C$, we have

$$\int 2\sin x + \cos x \; dx = 2(-\cos x) + \sin x + C = \sin x - 2\cos x + C.$$

So $\displaystyle\int_0^{\pi/6} 2\sin x + \cos x \; dx = [\sin x - 2\cos x]\Big|_0^{\pi/6}$

$= [\sin(\pi/6) - 2\cos(\pi/6)] - [\sin(0) - 2\cos(0)] = [(1/2) - 2(\sqrt{3}/2)] - [0 - 2(1)]$

$= 1/2 - \sqrt{3} + 2 = 5/2 - \sqrt{3}.$

3. $\displaystyle\int \frac{x^3 + x^2 - 3}{x^2} \; dx$

The integrand can be written more usefully as

$\dfrac{x^3 + x^2 - 3}{x^2} = x + 1 - 3x^{-2}.$ So we have

$$\int \frac{x^3 + x^2 - 3}{x^2} \; dx = \int x + 1 - 3x^{-2} \; dx$$

$$= \int x \; dx + \int 1 \; dx - 3\int x^{-2} \; dx = \frac{x^2}{2} + x - 3\frac{x^{-1}}{-1} + C$$

$$= \frac{1}{2}x^2 + x + 3x^{-1} + C \; .$$