Math 107H, Section 1

Quiz number 1 Solutions

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

1. Use sums-of-rectangles, using (a) left endpoints and (b) right endpoints, with $n = 4$, to estimate the area under the graph of the function $f(x) = \dfrac{1}{x+1}$, between $x = 0$ and $x = 2$. How far apart are your two estimates?

Cutting the interval $[0, 2]$ into four equal intervals, each will have length $1/2$. The left endpoints will be $0, 1/2, 1$, and $3/2$, so the left-endpoint estimate will be

$$\frac{1}{2}(f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{2})) = \frac{1}{2}(\frac{1}{1+0} + \frac{1}{1+[1/2]} + \frac{1}{1+1} + \frac{1}{1+[3/2]})$$
$$= \frac{1}{2}(1 + 1/(3/2) + 1/2 + 1/(5/2)) = \frac{1}{2}(1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5})$$
$$= \frac{1}{2}(\frac{30}{30} + \frac{20}{30} + \frac{15}{30} + \frac{12}{30})$$
$$= \frac{77}{60}$$

Similarly, the right endpoints are $1/2, 1, 3/2$, and 2, and so the right-endpoint estimate is

$$\frac{1}{2}(\frac{1}{1+(1/2)} + \frac{1}{1+1} + \frac{1}{1+[3/2]} + \frac{1}{1+2}) = \frac{1}{2}(\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}$$
$$= \frac{1}{2}(\frac{20}{30} + \frac{15}{30} + \frac{12}{30} + \frac{10}{30}) = \frac{57}{60} = \frac{19}{20} .$$

The difference (left) $-$ (right) is $\frac{1}{2}(1 - \frac{1}{3}) = \frac{1}{2}\frac{2}{3} = \frac{1}{3} = \frac{20}{30}$.