Math 107H, Section 1

Quiz number 10 Solutions

Show all work! How you get your answer is just as important, if not more important, than the answer itself.

12. Find the radius (radii?) of convergence of the following power series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^4 x^n}{n!}$

The coefficients of the power series are $a_n = \dfrac{n^4}{n!}$, and so, using the ratio test, we compute:

$$\left|\frac{a_{n+1}}{a_n}\right| = (\frac{(n+1)^4}{(n+1)!})/(\frac{n^4}{n!}) = \frac{(n+1)^4}{n^4} \cdot \frac{n!}{(n+1)!} = (1+\frac{1}{n})^4 \cdot \frac{1}{n+1},$$

which converges to $(1+0)^4 \cdot 0 = 0 = L$. So the radius of convergence of the power series is "$\dfrac{1}{0}$" $= \infty$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{5^n}(x-3)^n$

The coefficients of the power series are $a_n = \dfrac{n^2}{5^n}$, and so, using the ratio test, we compute:

$$\left|\frac{a_{n+1}}{a_n}\right| = (\frac{(n+1)^2}{5^{n+1}})/(\frac{n^2}{5^n}) = \frac{(n+1)^2}{n^2} \cdot \frac{5^n}{5^{n+1}} = (1+\frac{1}{n})^2 \cdot \frac{1}{5},$$

which converges to $(1+0)^2 \cdot \dfrac{1}{5} = \dfrac{1}{5} = L$. So the radius of convergence of the power series is $\dfrac{1}{L} = 5$.