Math 107H Exam 1 Practice Problems

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

Find each of the following integrals.

Note that " $\int_3^x f(t)\, dt + C$ " is not a sufficient computation of an antiderivative!

Some formulas of potential use can be found at the bottom of the last page of the exam.

1. (10 pts.) $\int (x+2)^{3/2}\, dx$
2. (15 pts.) $\int_0^{\pi/2} \sin^3 x\, dx$
3. (10 pts.) $\int \frac{x^2+x-3}{x^{1/2}}\, dx$
4. (15 pts.) $\int_0^1 e^{\sqrt{x}}\, dx$
5. (15 pts.) $\int \frac{dx}{(x+1)^2(x+4)}$
6. (15 pts.) $\int e^{-x}\sin(3x)\, dx$
7. (20 pts.) $\int (x^2+1)^{3/2}\, dx = \int \left(\sqrt{x^2+1}\,\right)^3 dx$

Some formulas:

$$\int \sec^n x\, dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$$

$$c^2 \int \frac{dy}{(y^2+c^2)^k} = \frac{1}{(2k-2)} \cdot \frac{y}{(y^2+c^2)^{k-1}} + \frac{(2k-3)}{(2k-2)}\int \frac{dy}{(y^2+c^2)^{k-1}}$$

1. (10 pts. each) Find the following integrals:

   (a): $\int_1^4 x^2 \ln x\, dx$ (b): $\int \sin^2 x \cos^3 x\, dx$

2. (10 pts. each) When you apply the appropriate trigonometric substitutions, what do the following integrals become?

   (a): $\int \frac{\sqrt{4-x^2}}{x^2}\, dx$ (b): $\int \frac{x^2}{\sqrt{4x^2+9}}\, dx$

In problems 1 through 4, find each of the following integrals.

1. (10 pts.) $\int_2^3 (3x-2)^{\frac{5}{4}}\, dx$
2. (15 pts.) $\int x^{\frac{1}{3}} \ln x\, dx$
3. (10 pts.) $\int_0^{\pi/2} \cos^3 x\, dx$

4. (15 pts.) $\displaystyle\int \frac{dx}{1+\sqrt{x}}$

6. (15 pts.) Use the method of partial fractions to express the function $f(x) = \dfrac{1}{(x+2)(x+5)}$ as a sum of more elementary rational functions. Then **with no more work** use this answer to express $g(x) = \dfrac{1}{(x^2+2)(x^2+5)}$ as a sum of elementary rational functions, and find the integral

$$\int \frac{1}{(x^2+2)(x^2+5)}\, dx \ .$$

6. (15 pts.) Recall that if a function f has second derivative satisfying $|f''(x)| \leq M$ for every x in the interval $[a, b]$, then the error E_n in approximating the integral $\int_a^b f(x)\, dx$ using the trapezoidal rule using n equal subintervals is at most

$$M\frac{(b-a)^3}{12n^2}$$

Based on this, how many subintervals should we divide the interval $[2, 5]$ into in order to be sure to approximate the integral $\int_2^5 x \ln x \, dx$ with an error of less than $\frac{1}{100}$?