Math 107H

A not-so-short table of integrals

First building blocks:

$$\int x^n\ dx = \frac{x^{n+1}}{n+1} + C \text{ (provided } n \neq -1) \qquad \int \frac{1}{x}\ dx = \ln|x| + C \qquad \int e^{ax}\ dx = \frac{e^{ax}}{a} + C$$

$$\int \sin(kx)\ dx = \frac{-\cos(kx)}{k} + C \qquad \int \cos(kx)\ dx = \frac{\sin(kx)}{k} + C$$

$$\int \sec^2 x\ dx = \tan x + C \qquad \int \sec x\ dx = \ln|\sec x + \tan x| + C \qquad \int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc^2 x\ dx = -\cot x + C \qquad \int \csc x\ dx = -\ln|\csc x + \cot x| + C \qquad \int \csc x \cot x\ dx = -\csc x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C \qquad \int \cot x\ dx = \ln|\sin x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{Arcsin}(\tfrac{x}{a}) + c \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\ \text{Arctan}(\tfrac{x}{a}) + c \qquad \int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\text{Arcsec}(\tfrac{x}{a}) + c$$

Reduction formulas:

$$\int x^n e^{ax}\ dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\ dx \qquad \text{[by parts: } u = x^n]$$

$$\int x^n \sin(bx)\ dx = \frac{-1}{b}x^n\cos(bx) + \frac{n}{b}\int x^{n-1}\cos(bx)\ dx \qquad \text{[by parts: } u = x^n]$$

$$\int x^n \cos(bx)\ dx = \frac{1}{b}x^n\sin(bx) - \frac{n}{b}\int x^{n-1}\sin(bx)\ dx \qquad \text{[by parts: } u = x^n]$$

$$n\int \sin^n x\ dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\ dx \qquad \text{[by parts: } dv = \sin x\ dx]$$

$$n\int \cos^n x\ dx = \cos^{n-1} x\sin x - (n-1)\int \cos^{n-2} x\ dx \qquad \text{[by parts: } dv = \cos x\ dx]$$

$$(n-1)\int \sec^n x\ dx = \sec^{n-2} x\tan x + (n-2)\int \sec^{n-2} x\ dx \qquad \text{[by parts: } dv = \sec^2 x\ dx]$$

$$\int \tan^n x\ dx = [1/(n-1)]\tan^{n-1} x - \int \tan^{n-2} x\ dx \qquad \text{[by } u\text{-subs: } \tan^2 x = \sec^2 x - 1]$$

$$\int \cot^n x\ dx = -\int \tan^n u\ du\ \Big|_{u=\frac{\pi}{2}-x} \qquad \int \csc^n x\ dx = -\int \sec^n u\ du\ \Big|_{u=\frac{\pi}{2}-x}$$

Special forms:

$$\int e^{ax}\sin bx\ dx = \frac{1}{a^2+b^2}[ae^{ax}\sin(bx) - be^{ax}\cos(bx)] + C \text{ [Or: remember the basic form, and } \tfrac{d}{dx}\text{ !]}$$

$$\int e^{ax}\cos bx\ dx = \frac{1}{a^2+b^2}[be^{ax}\sin(bx) + ae^{ax}\cos(bx)] + C \text{ [Or: remember the basic form, and } \tfrac{d}{dx}\text{ !]}$$

$\sin(ax)\sin(bx) = (1/2)[\cos(a-b)x) - \cos((a+b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\sin(ax)\cos(bx) = (1/2)[\sin(a+b)x) + \sin((a-b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\cos(ax)\cos(bx) = (1/2)[\cos(a-b)x) + \cos((a+b)x)]$; integrate <u>that</u> instead! [Or: by parts, twice!]

$\int \sin^n x\cos^m x\ dx$:

- if n or m odd, set aside one of $\sin x\ dx$ or $\cos x\ dx$ and convert the rest,
  - using $\sin^2 x + \cos^2 x = 1$
- if both n, m even, convert all to powers of $\sin x$ and use reduction formula

$\int \sec^n x\tan^m x\ dx = \int \dfrac{\sin^m x}{\cos^{n+m} x}\ dx$:

- if n even, keep two, convert using $\sec^2 x = \tan^2 x + 1$, and u-sub:
  - integrand is (powers of $\tan x$)($\sec^2 x\ dx$)
- if m odd, keep one each, using $\tan^2 x = \sec^2 x - 1$, and u-subs:
  - integrand is (powers of $\sec x$)($\sec x\tan x\ dx$)
- otherwise, convert $\tan x$'s to $\sec x$'s and use reduction formula.

$$\int \frac{\cos^m x}{\sin^{n+m} x}\ dx = \int \csc^n x\cot^m x\ dx = -\int \sec^n u\tan^m u\ du\ \Big|_{u=\frac{\pi}{2}-x}$$

$\displaystyle\int \frac{dx}{\sin^n x\cos^m x} = \int \frac{\sin x\ dx}{\sin^{n+1} x\cos^m x} = \int \frac{\cos x\ dx}{\sin^n x\cos^{m+1} x}$; n, m both even is a <u>bear</u>...

$\displaystyle\frac{1}{\sin^{2n} x\cos^{2m} x} = \frac{\cos^{2k} x \text{ or } \sin^{2k} x}{(\sin x\cos x)^{2r}} = \frac{[\frac{1}{2}(1 \pm \cos(2x)]^k}{[\frac{1}{2}\sin(2x)]^r}$; u-sub ($u = 2x$) and expand out... !