Math 107H Fall 2010
Integration: Don't take a function at face value!

An integral that we can't handle, as written, can sometimes be transformed into something we can. Basically, we can do anything we want to the integrand, so long as we don't <u>change</u> the actual function!

Multiply the function out!

Example: $\int (e^x+3)^2 \; dx = \int (e^x)^2 + 6e^x + 9 \; dx = \int e^{2x} + 6e^x + 9 \; dx = \cdots$

Pull fractions apart!

Example: $\int \frac{x+1}{x^3} \; \mathrm{d}x = \int x^{-2} + x^{-3} \; \mathrm{d}x = \cdots$

Put fractions together!

Example: $\int \frac{x}{(x+1)^2+1} + \frac{1}{(x+1)^2+1} \; dx = \int \frac{x+1}{(x+1)^2+1} \; dx = \int \frac{u \; du}{u^2+1}\bigg|_{u=x+1} = \cdots$

Do polynomial long division!

Example: $\int \frac{x^3}{x^2-1} \; \mathrm{d}x = \int x + \frac{x}{x^2-1} \; \mathrm{d}x = \cdots$

Use trig identities!

$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $\sin(2x) = 2\sin x \cos x$, $\frac{\tan x}{\sec x} = \sin x$, etc.

Example: $\int \tan x \sin x \; dx = \int \frac{\sin^2 x}{\cos x} \; dx = \int \frac{1-\cos^2 x}{\cos x} \; dx = \int \sec x - \cos x \; dx = \cdots$

Add zero!

Example: $$\int \frac{x}{(x+1)^2+1} = \int \frac{x}{(x+1)^2+1} + \frac{1}{(x+1)^2+1} - \frac{1}{(x+1)^2+1} \; dx$$
$$= \int \frac{x+1}{(x+1)^2+1} \; dx - \int \frac{1}{(x+1)^2+1} \; dx = \cdots$$

Multiply by one!

Example: $$\int \frac{1}{\sin x + 1} \; dx = \int \frac{1}{\sin x + 1} \frac{1-\sin x}{1-\sin x} \; dx = \frac{1-\sin x}{1-\sin^2 x} \; dx = \frac{1-\sin x}{\cos^2 x} \; dx$$
$$= \int \sec^2 x - \sec x \tan x \; dx = \cdots$$

Example: $\int \sec x \; \mathrm{d}x = \int \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} \; \mathrm{d}x = \cdots$

Complete the square!

$ax^2 + bx + c = a(x^2 + rx) + c = a(x + r/2)^2 + (c - (r/2)^2)$

Example: $\int \frac{1}{x^2+2x+2} \; dx = \int \frac{1}{(x+1)^2+1} \; dx = \int \frac{1}{u^2+1} \; du\bigg|_{u=x+1} = \cdots$