Math 107H Practice Problems for Exam 2

Show all work. How you get your answer is just as important, if not more important, than the answer itself.

1. Find the volume of the region obtained by revolving the region under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \pi$ around the y-axis.

2. Find the improper integral $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^3}\, dx$.

3. Determine the convergence or divergence of the following sequences:

(a) $a_n = \dfrac{n^3 + 6n^2 \ln n - 1}{2 - 3n^3}$ (b) $b_n = \dfrac{n^{n+\frac{1}{n}}}{(n+3)^n}$

4. Determine the convergence or divergence of the following series:

(a) $\displaystyle\sum_{n=2}^\infty \frac{1}{(n-1)(\ln n)^{2/3}}$ [Hint: limit compare, <u>then</u> integral...]

(b) $\displaystyle\sum_{n=0}^\infty \frac{6n}{(1-n^2)^2}$

5. Determine the convergence or divergence of the following series:

(a) $\displaystyle\sum_{n=1}^\infty \frac{(n-1)!}{2^n n^3}$ (b) $\displaystyle\sum_{n=0}^\infty \frac{n2^{2n+1}}{9^n+1}$

6. Set up, **but do not evaluate**, the integral which will compute the arclength of the graph of $y = x\sqrt{1+x^2}$ from $x = 0$ to $x = 3$.

7. Cesium-137, denoted Cs_{137}, is a radioactive substance with a half-life of 30 years. That is, if $C(t)$ represents the amount of Cs_{137} in a sample after t years, then

$$C(30) = \frac{1}{2}C(0)\ .$$

If we start with a 4 gram sample of Cs_{137}, how much Cs_{137} will remain after 10 years?

8. Find the (implicit) solutions to the differential equation

$$\frac{dy}{dt} = \frac{te^{y+t}}{y}$$