Math 107 Calculus II Name: ______________________

Final Exam May 2, 2017

Circle the name of your instructor and, in the appropriate column, the name of your recitation leader. The second row is the time of your lecture.

Radu	Ledder	Marley	Walker	De Silva	West
9:30 TR	10:30 MW	12:30 MW	8:30 MW	10:30 MW	6:30 TR
Falahola	Moeller	Falahola	Moeller	De Silva	West
Wright	Nasr	Carlson	McMillon		
Becklin	McMillon	Becklin	Nasr		
White	Awadalla	Awadalla	Carlson		
	White		Tomlinson		

Instructions

- Turn off all communication devices.
- During the exam, you may not use any calculators or electronic devices of any sort. Nor may you use any notes, texts, references, etc.
- To receive full credit for a problem, you must provide a correct answer **and a sufficient amount of work** so that it can be determined how you arrived at your answer.
- Be sure your copy of this exam has 10 pages (including this page) with 11 problems.
- Good luck!

Some commonly used Taylor series:

$$\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad \cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^n}{n} \qquad \ln(x) = \sum_{n=1}^{\infty}(-1)^{n+1} \frac{(x-1)^n}{n}$$

Problem	Points	Score
1	16	
2	12	
3	16	
4	18	
5	18	
6	12	
7	10	
8	20	
9	14	
10	6	
11	20	
12	14	
13	24	
Total	200	

1. Consider the definite integral

$$\int_2^6 \frac{x}{\sqrt{x-2}}dx.$$

   (a) (6 points) Explain why it is an improper integral.

   (b) (10 points) If it converges, find its exact value. If it diverges, justify why it diverges.

2. Let f and g be continuous functions. Circle the appropriate statement below:

   (a) (4 points) Assume $0 \leq f(x) \leq g(x)$ and $\int_1^\infty f(x)\,dx$ converges.

   Then $\int_1^\infty g(x)\,dx$ **Converges** **Diverges** **We cannot conclude**

   (b) (4 points) Assume $0 \leq f(x) \leq g(x)$ and $\int_1^\infty f(x)\,dx$ diverges.

   Then $\int_1^\infty g(x)\,dx$ **Converges** **Diverges** **We cannot conclude**

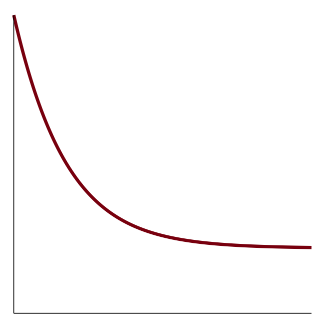
   (c) (4 points) Assume $0 \leq f(x) \leq g(x)$ and $\int_1^\infty g(x)\,dx$ converges.

   Then $\int_1^\infty f(x)\,dx$ **Converges** **Diverges** **We cannot conclude**

3. (16 points)
   Consider the definite integral

$$\int_0^1 \sqrt{e^{-2x}+3}\,dx.$$

   The graph of $y = \sqrt{e^{-2x}+3}$ is pictured to the right.



   (a) (8 points) Use the trapezoidal rule with $n = 2$ subdivisions to approximate the above integral. (Your answer can involve expressions like $\sqrt{e^{-\frac{3}{4}}+3}$ — you do not need to give a decimal approximation.)

   (b) (8 points) Decide if the approximation obtained in part (a) is an overestimate or underestimate of the exact value, and explain why.

4. In parts (a), (b) and (c) below, do the following:

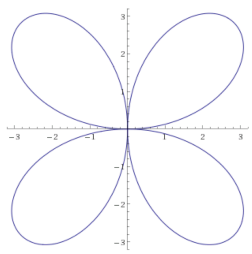
- if it is a convergent geometric series, **find the sum of the series**, or
- if it is a divergent geometric series, write "divergent geometric series", or
- if it is not a geometric series, write "not a geometric series".

(a) (6 points) $5 + \frac{25}{2} + \frac{125}{4} + \frac{625}{8} + \cdots$:

(b) (6 points) $\sum_{n=1}^{\infty} \frac{1}{n^2}$:

(c) (6 points) $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$:

5. Consider the four-leaf petal defined by $r = 4\sin(2\theta)$:



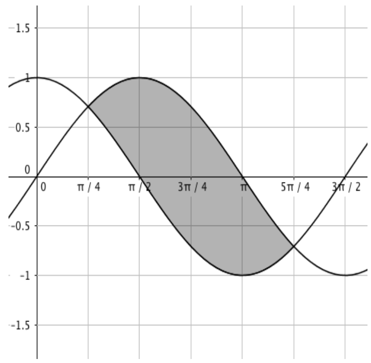
(a) (12 points) Find the area of one of the petals. *Tip:* $\int \sin^2(x)\,dx = \frac{x - \sin(x)\cos(x)}{2} + C$.

(b) (6 points) Find the polar coordinates of each of the points where this curve meets the line $y = x$.

6. (12 points) Evaluate $\int_0^{\infty} xe^{-x}\,dx$.

7. (10 points) Use a trigonometric substitution to simplify $\int \sqrt{4 - x^2}\,dx$ so that there is no square root in the integrand. You do NOT need to evaluate the simplified integral.

8. Consider the region bounded by $y = \cos(x)$ and $y = \sin(x)$, for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$, as pictured.



(a) (8 points) Set up, but do NOT evaluate, one or more definite integrals that give the perimeter of the shaded region. (Recall that the "perimeter" is the length of the boundary of a region.)

(b) (12 points) Set up, but do NOT evaluate, one or more definite integrals that give the volume of the solid obtained by rotating the shaded region about the line $y = 1$.

9. (14 points) In the spaces below, arrange the four functions

$$f(x) = 1 + x\sin\left(\frac{x}{2}\right), \quad g(x) = \frac{1}{1-x^2}, \quad h(x) = \cos(x), \quad \text{and} \quad j(x) = e^{-x^2}$$

in increasing order, from smallest to largest, for values of x near $x = 0$. Justify your answer by using the degree two Taylor polynomial approximations of these functions. (Recall that there is table of common Taylor series on the front page of this exam.)

10. (6 points) Given the following information about a function $g(x)$,

$$g(2) = -3, \; g'(2) = 5, \; g''(2) = -7, \; g^{(3)}(2) = 9, \; \text{and } g^{(4)}(2) = -11,$$

write down the fourth degree Taylor polynomial that approximates $g(x)$ near $x = 2$.

11. The Taylor series about $x = 0$ of a certain function $f(x)$ is

$$\sum_{n=1}^{\infty} \frac{3^n}{n} x^{2n} = 3x^2 + \frac{9}{2}x^4 + \frac{27}{3}x^6 + \cdots .$$

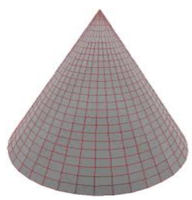
(a) (6 points) Find $f^{(8)}(0)$.

(b) (14 points) Find the radius of convergence of the series. *You do not need to determine convergence at the endpoints.*

12. (14 points) In the far away land of Aksarben, ancient people built a solid stone temple in the shape of a perfect cone, as pictured below. The cone is 100 feet tall with a base radius of 25 feet. The stone used to build it weighs 320 pounds/ft^3. Assuming that the only work involved was to lift the stone blocks up from ground level, how much work was needed to build the temple? Set up a definite integral that gives that answer, but do NOT evaluate it.

    To receive full credit, be sure to:

    - define the variable(s) you introduce and label them on a diagram,
    - indicate clearly how you found the weight (force) of a typical slice of the cone,
    - specify the limits of integration, and
    - include units.



13. Determine whether each of the following series converges or diverges, using any valid method. **Be sure to make it clear which test(s) you are using and to indicate why they apply.**

    (a) (12 points) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{3n^2+1}$

    (b) (12 points) $\displaystyle\sum_{n=2}^{\infty}\frac{3n+1}{\sqrt{n^4-1}}$