A Model for COVID-19 with Limited Vaccination

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Aim of the Research

- Study the impact of key interacting factors:
  - social distancing and mask use
  - *testing [included, but not studied in this talk]*
  - vaccination distribution rate
  - vaccination efficacy
  - vaccination acceptance

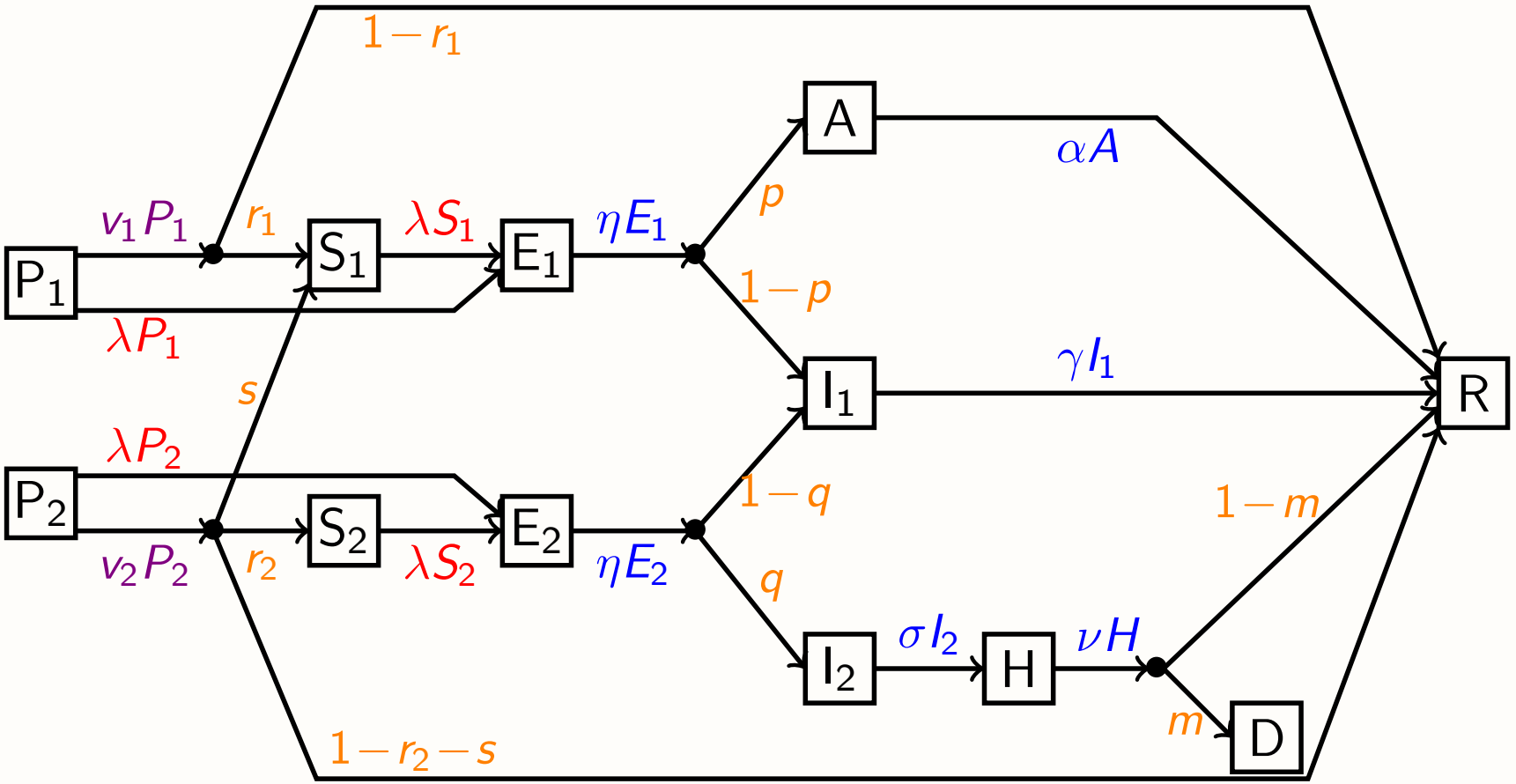
Key Requirements Beyond SEIR

- Multiple Infectious Classes:
  - Symptomatic
  - Asymptomatic
    - shorter duration, lower infectivity, no testing
  - Hospitalized
    - possible mortality, no infectivity
- Multiple Interventions
  - Masks and Physical Distancing
    - reduced infectivity for unconfirmed cases
  - Testing and Isolation
    - reduced infectivity for confirmed cases
  - Vaccination (*infection-blocking and/or disease-blocking*)
    - not everyone accepts, not always effective
    - limited supply

The PSEAIHRD Model (expanded SEIR)

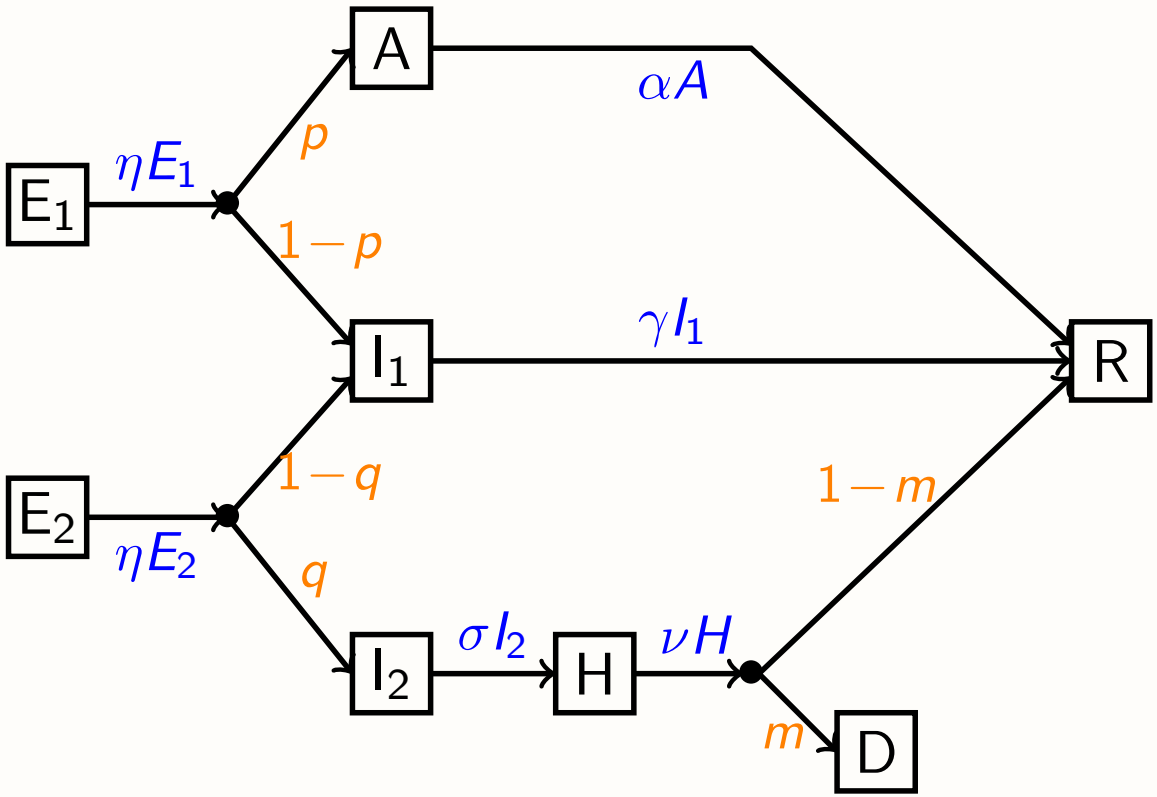
- Susceptible Classes
  - **P**re-vaccinated
  - **S**usceptible
- Exposed (latent) Classes
  - **E**xposed
- Infectious Classes
  - **A**symptomatic
  - **I**nfectious (symptomatic)
  - **H**ospitalized
- Removed Classes
  - **R**ecovered (or vaccinated)
  - **D**eceased

The PSEAIHRD Model

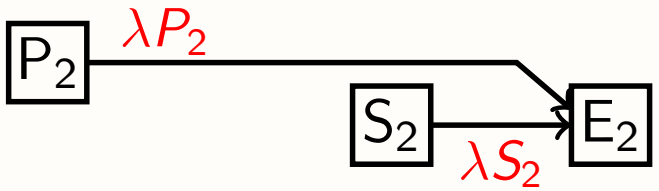


transitions, transmissions, vaccination, divergent paths

The PSEAIHRD Model – Transitions



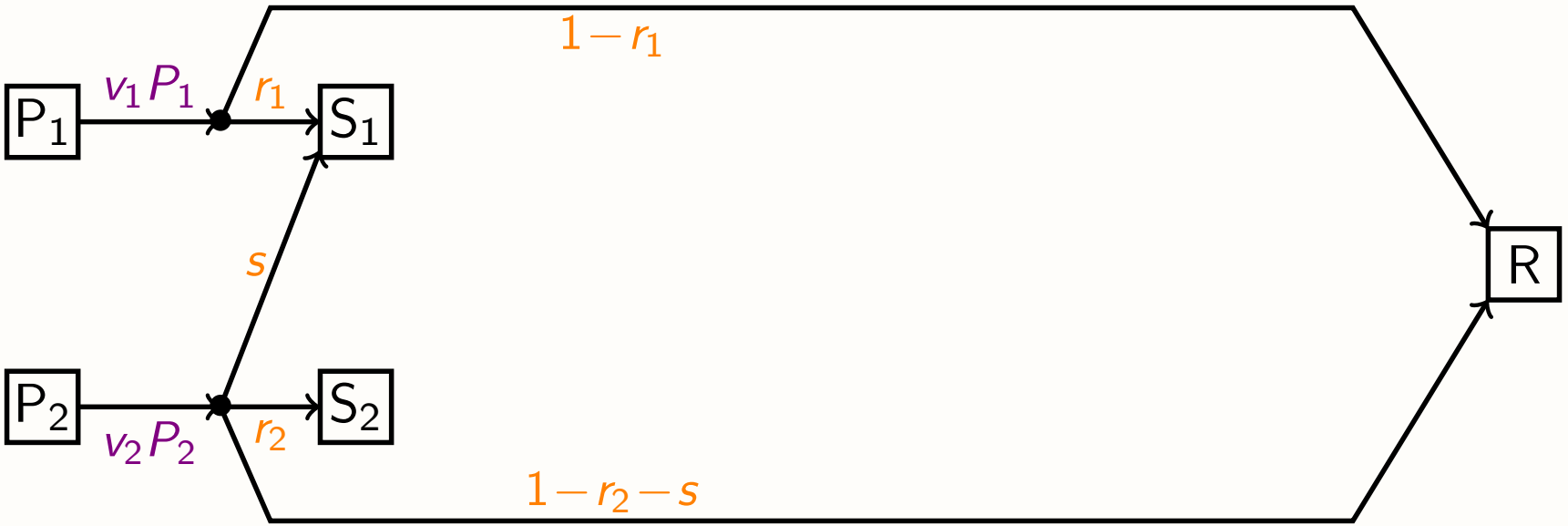
The PSEAIHRD Model – Transmissions



$$\lambda = \beta X, \quad X = f_c(p_c I + p_{ca} A) + \delta[(1 - p_c)I + f_a(1 - p_{ca})A].$$

- p_c and p_{ca} are the fractions of confirmed cases for symptomatic and asymptomatic infectives.
- f_c, f_a are the infectivities of confirmed cases, and asymptomatic cases, relative to an unconfirmed symptomatic infective.
- δ is a 'contact factor' that incorporates physical distancing and mask use for unconfirmed cases.

The PSEAIHRD Model – Vaccination

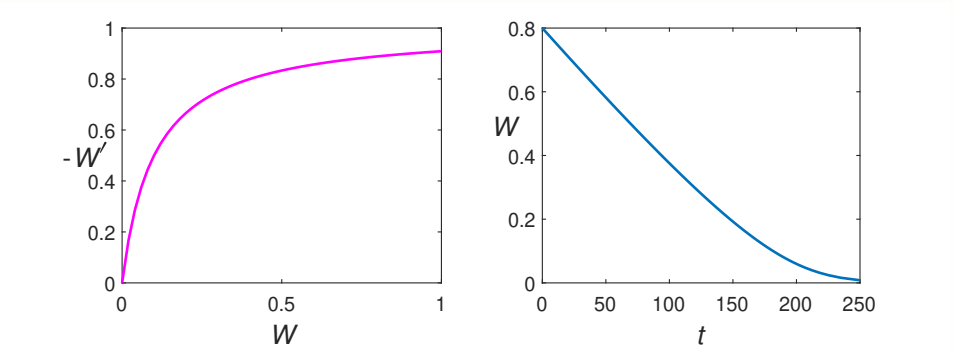


- r_i are the probabilities of vaccine failure.
- s is the probability that the vaccine fails to stop infection.
- v_i are rate 'constants' that depend on the status of the vaccination program.

Base Vaccination Model

$$W' = -\frac{V\,W}{K+W}, \qquad W(0) = W_0 = 1 - d.$$

- W is the fraction of people who want vaccination.
  - d is the population fraction of vaccine dissenters.
- Finding recipients for a vaccinator is like finding substrate for a biochemical enzyme.
  - V is the supply-limited maximum vaccination rate (population fraction per day).
  - K is the semi-saturation level.



Two-Class Vaccination Model

- Suppose a fraction τ of W is high-risk.

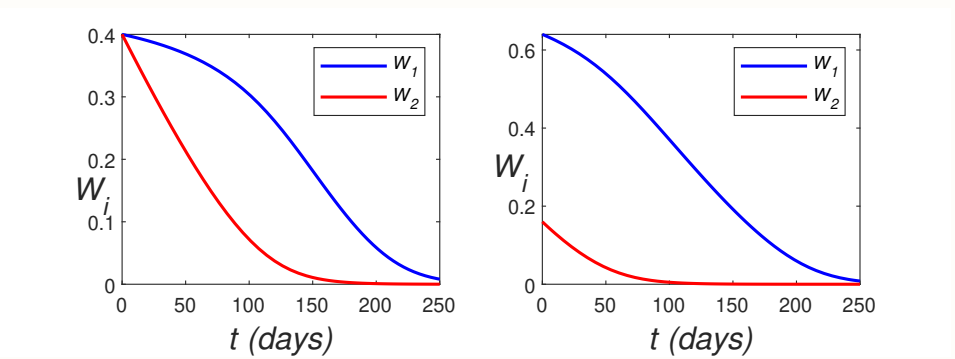
$$W_2(0) = \tau W_0, \quad W_1(0) = (1-\tau)W_0.$$

- Assume the same model for high-risk as for the whole group.

$$W_2' = -\frac{V}{K+W_2} W_2 \equiv -v(W_2)W_2, \quad v(W_2) = \frac{V}{K+W_2}.$$

  - Then

$$W_1' = W' - W_2' = \cdots = -u(W)v(W_2)W_1, \quad u(W) = \frac{K}{K+W}.$$



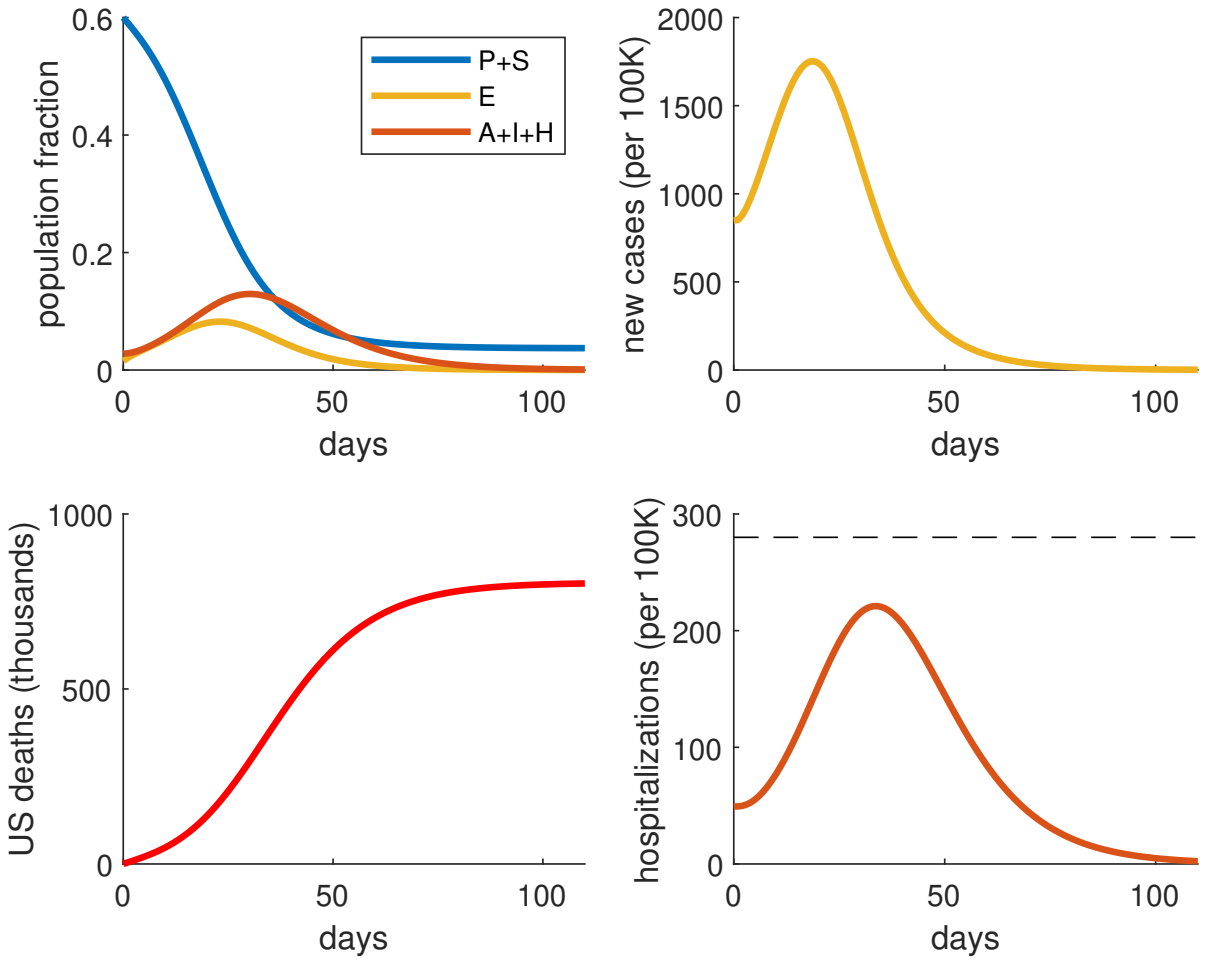
PSEAIHRD Vaccination Details

- We need v_i for
$$P_i' = -v_i P_i - \lambda P_i.$$
- We have
$$W_1' = -u(W)v(W_2)W_1, \quad W_2' = -v(W_2)W_2,$$
$$v(W_2) = \frac{V}{K + W_2}, \quad u(W) = \frac{K}{K + W}.$$
- Therefore
$$v_2 = v(W_2), \quad v_1 = u(W)v(W_2).$$
- We need only couple the W and W_2 equations to the PSEAIHRD system.

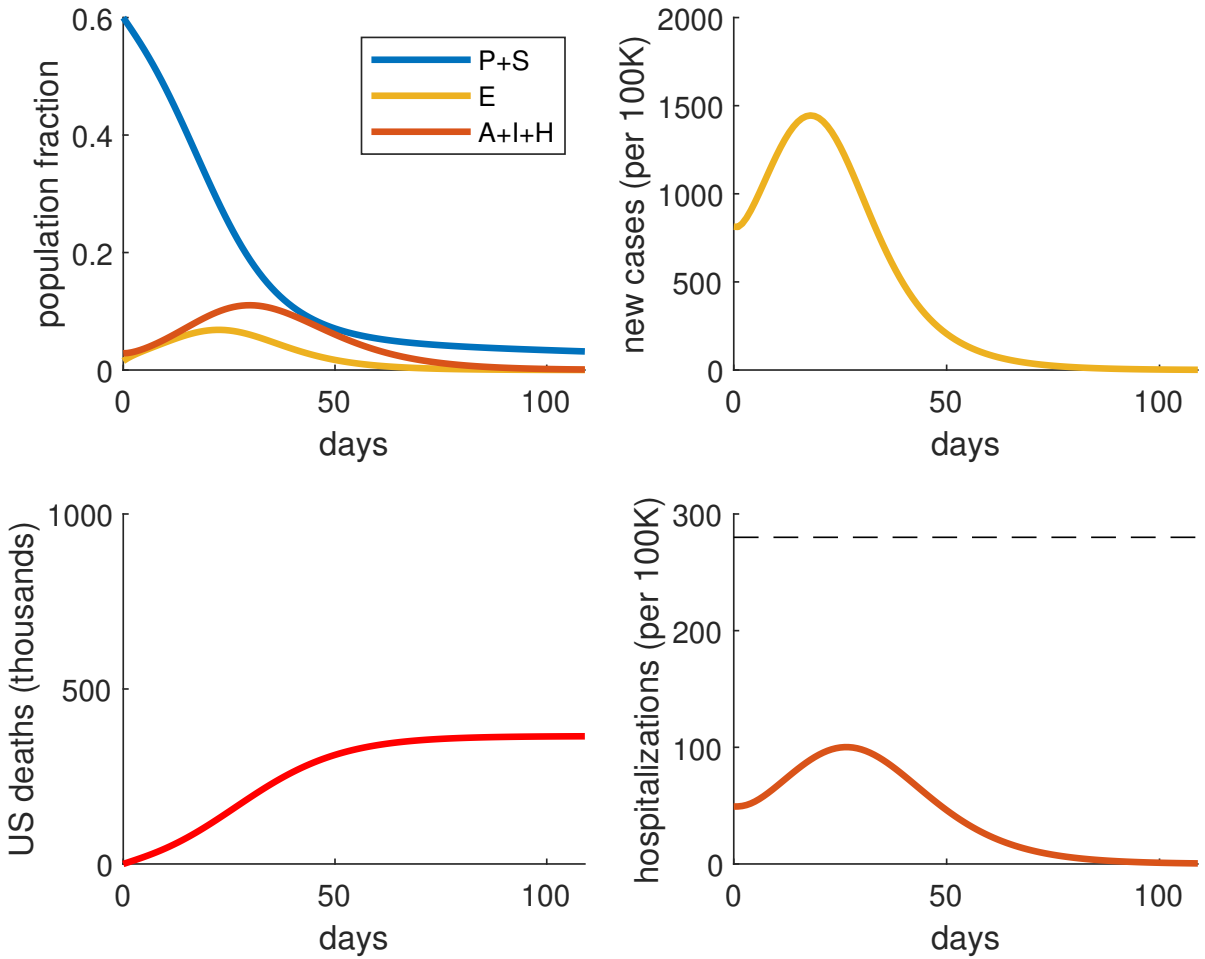
Key Fixed Parameter Values

- Types of Cases (pre-vaccination):
  - 60% of cases symptomatic
  - 20% of cases confirmed
  - 8% of confirmed cases require hospitalization (1.6% of total)
  - 25% mortality for hospitalized patients (0.4% of total)
- Initial State
  - 20% of the population is at high risk.
  - 60% of people initially susceptible (S and P)
  - Infectious classes calculated from assumption of 4000 US deaths per day.
- Vaccination Parameters
  - maximum rate $V = 0.005$

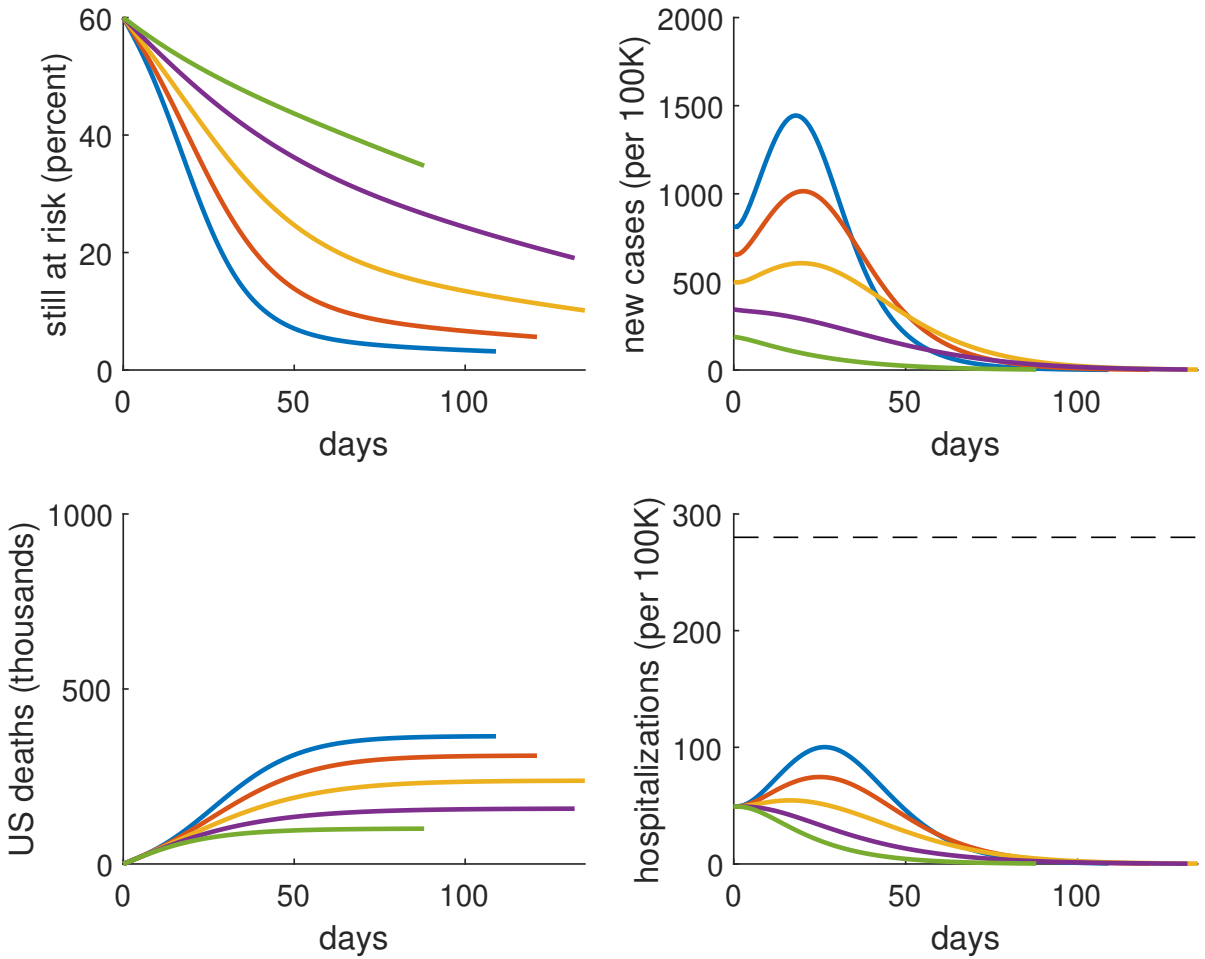
Base Scenario: $\delta = 1,\ V = 0,\ \mathcal{R}_0 = 6$



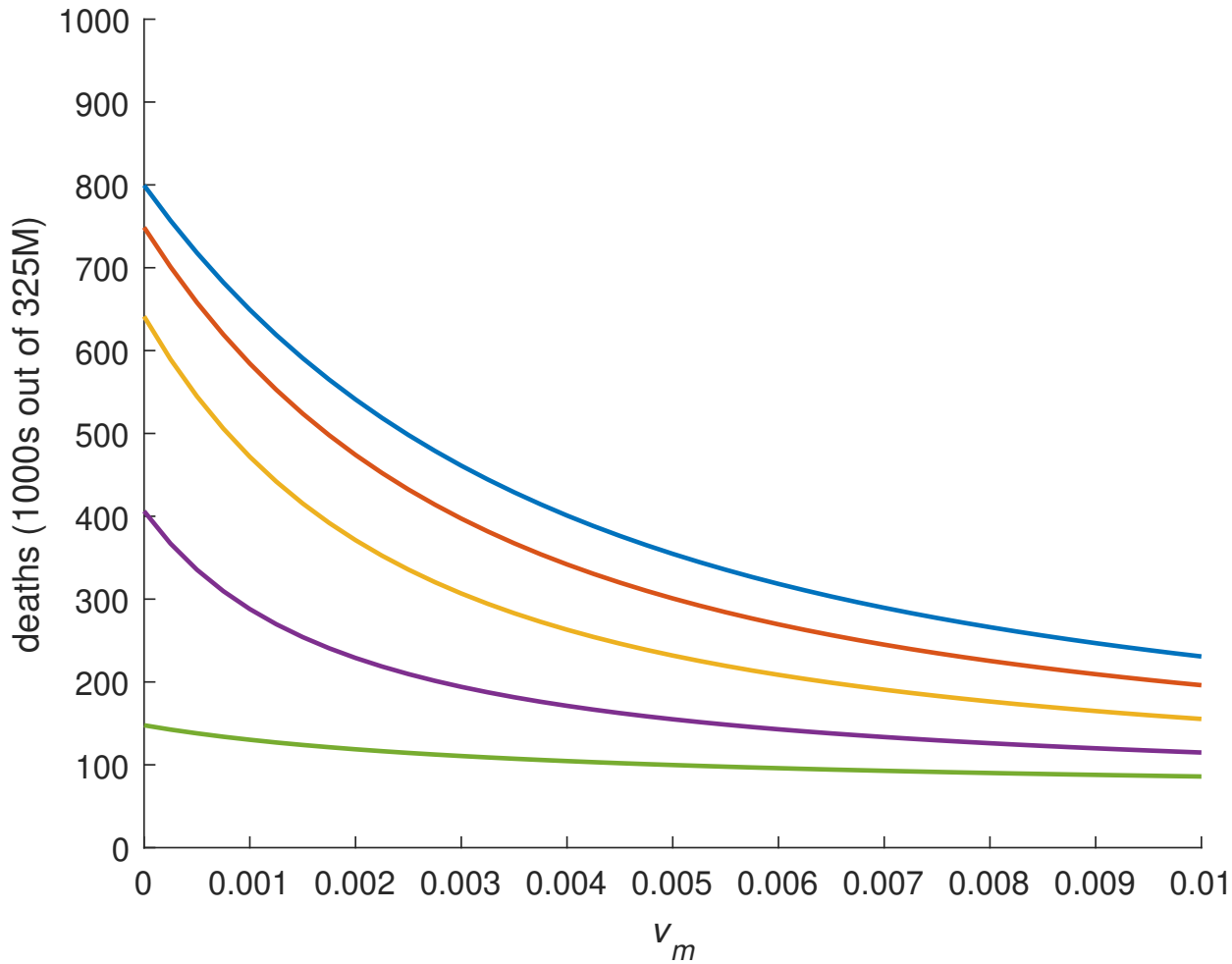
Vaccination Scenario: $\delta = 1$, $V = 0.005$, $\mathcal{R}_0 = 6$



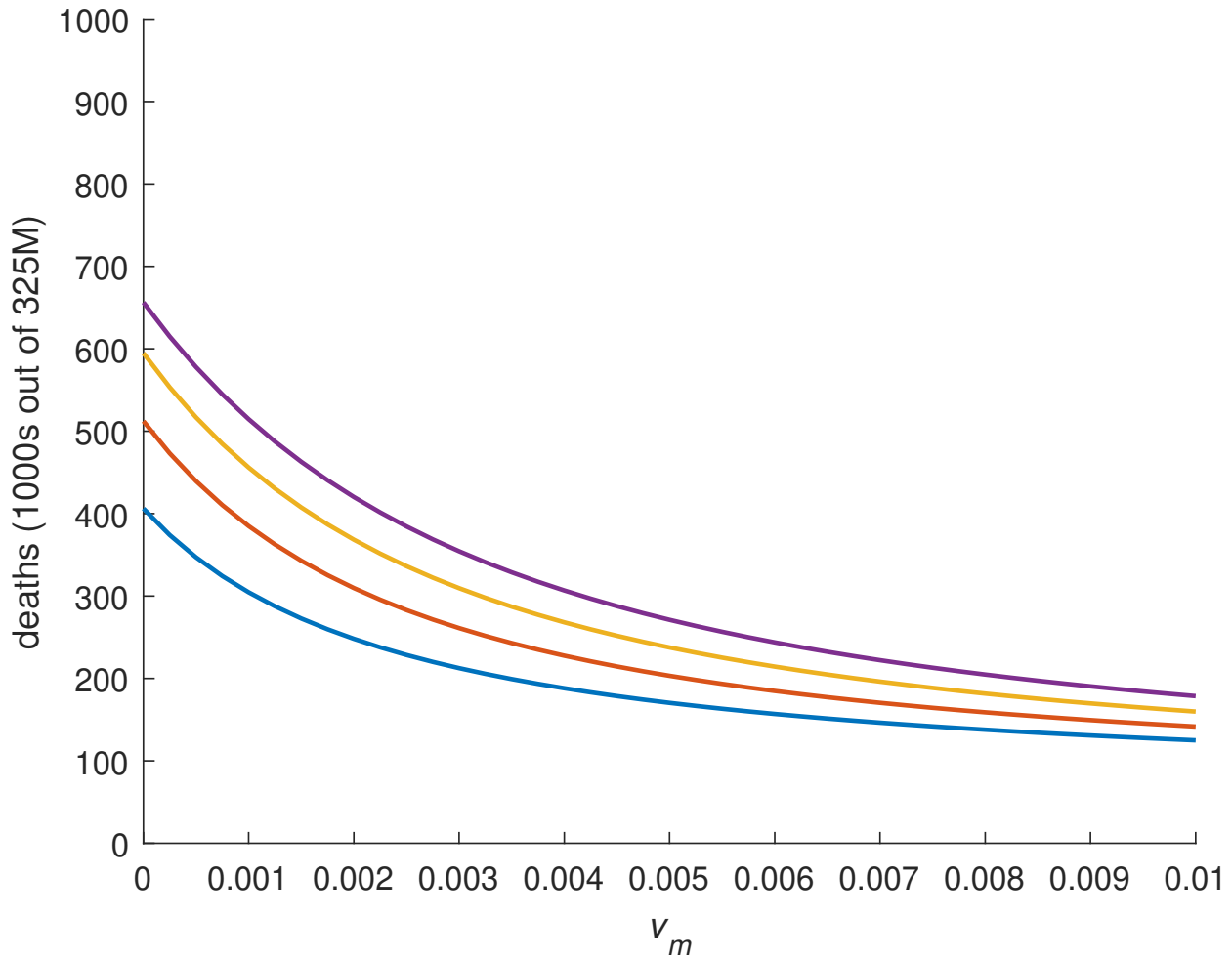
$V = 0.005$; $\delta = 1, 0.8, 0.6, 0.4, 0.2$ (bl.–gr., gold default)



D_∞ vs V ; $\delta = 1, 0.8, 0.6, 0.4, 0.2$ (bl–gr)



D_∞ vs V ; $R_0 = 6, 7, 8, 9$ (bl–vi)



Parameters That Matter Less

- The vaccine failure fraction r only matters if it is more than 30%.
- The partial failure fraction s does not matter.
- The dissension fraction d only matters if it is more than 50%.
  - Our scenarios assume 40% are already immune.

Resources

- See **https://www.math.unl.edu/SIR-modeling** for
  - Details on a classroom activity, including directions for an online implementation;
  - SIR and SEIR teaching modules using Excel and Matlab;
  - Links to some useful resources.
- See **https://www.math.unl.edu/covid-module** for COVID-19 teaching modules.
- Ledder and Homp, Using a COVID-19 model in various classroom settings to assess effects of interventions, PRIMUS 2021
  https://www.tandfonline.com/doi/full/10.1080/10511970.2020.1861143
- **Shoot me an email to receive updates or offer feedback!**
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